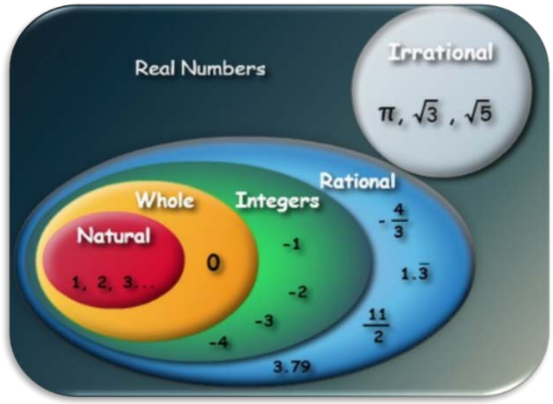
1.1-1.4 Properties of Real Numbers and Order of Operations

The Structure of the Set of Real Numbers:



set – a collection of objects called **elements** or **members**, usually having the same property

Notation:
roster notation – the elements of a set are listed in braces { } and separated by commas

Examples:
$\{1,2,3\}$ - **finite** set
$\mathbb{N} = \{1,2,3,\dots\}$ - **natural** or **counting** numbers (**infinite** set)
$\mathbb{W} = \{0,1,2,3,\dots\}$ - **whole** numbers
$\mathbb{Z} = \{\dots,-2,-1,0,1,2,\dots\} = \{0,\pm1,\pm2,\dots\}$ - **integers**

Notation:
$5 \in \mathbb{N}$ means 5 **is an element** of $\mathbb{N}$
$-1 \notin \mathbb{N}$ means -1 **is not an element** of $\mathbb{N}$
$\emptyset$ or { } denotes an **empty** (or **null**) set - the set that contains no elements

set-builder notation – the elements are determined by listing a property (or properties) that they need to satisfy

$\{x \in S \mid x \text{ has property } P\}$

the set of *all elements x from the set S* *such that* *x has a given property P*

Examples:

$\mathbb{Q} = \left\{ \frac{p}{q} \,\middle|\, p,q \in \mathbb{Z},\ q \neq 0 \right\}$ - **rational** numbers (any fraction, finite decimal, or infinite but repeating decimal)
$\mathbb{IQ} = \{x \mid x \notin \mathbb{Q}\}$ - **irrational** numbers (any infinite but non-repeating decimal, root of a prime number, constant π or e)
$\mathbb{Z}_- = \{x \mid x \in \mathbb{Z} \text{ and } x < 0\} = \{x \in \mathbb{Z} \mid x < 0\}$ - negative integers
$\mathbb{R} = \{x \mid \}$ - **real** numbers (numbers that correspond to any point on a number line – those are called **coordinates**)

Example 1: Prove that the following numbers are rational:
a) $\mathbf{1.\overline{5}}$ b) $\mathbf{0.23\overline{41}}$

opposite (additive inverse) – a number with a reverse sign; ex. $\boldsymbol{a}$ and $-\boldsymbol{a}$

reciprocal (multiplicative inverse) – a number with switched the numerator with the denominator; ex. $\boldsymbol{a}$ and $\boldsymbol{\frac{1}{a}}$

absolute value – a distance of a number from zero; $|a| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$

so **absolute value** of a number is always **positive!**

Practice:

1. Evaluate:

a) $-|-5| =$

b) $|2-6| - |-7| - |2| =$

2. Place one of the signs $<, >, \leq, \geq, =$ to make the statement true.

$-5 \quad\quad -7$ $\quad\quad$ $|7| \quad |-7|$ $\quad\quad$ $-|5| \quad |-5|$

$-|0| \quad |-0|$ $\quad\quad$ $|x| \quad x$ $\quad\quad$ $0 \quad |-x|$

Interval Notation:

Example 2: Complete the table.

set-builder notation	graph	interval notation
$\{x \mid x > 2\}$		
$\{x \mid x \leq -2\}$		
$\{x \mid -1 \leq x \leq 2\}$		
		
		
		$(-6,7)$
		

Order on the Number Line:

Match the following numbers with the letters on the number line: $-2,\ 3, -\pi,\ 5,\ \pi$

A B C D E

Observations:
- numbers are getting larger and larger towards the ....
- The arrow on the number line indicates the ....

How to find the **distance** between two given quantities on the number line?

5 and 8 .... -7 and -11 .... ***a*** and ***b*** ....

Practice:

3. Using the following graph, find

a) the total profit or loss for the years 2007 through 2010,

b) the average profit or loss per year for the years 2007 through 2010,

c) the difference between the profit or loss in 2010 and that in 2009,

d) the difference between the profit or loss in 2008 and that in 2007,

Order of Operations:

Recall: **sign rule** (for multiplication, division, or double sign)

$$\left.\begin{matrix} + & + \\ - & - \\ + & - \\ - & + \end{matrix}\right\} = \begin{matrix} + \\ + \\ - \\ - \end{matrix}$$

B brackets grouping signs (division bar, absolute value)

E exponents

D M ÷ · left to right

A S + − left to right

exponent n

$a^n = \underbrace{a \cdot a \cdot \ldots \cdot a}_{n\ times}$ so $(-3)^2 = (-3)(-3) = 9$ but $-3^2 = -3 \cdot 3 = -9$

power base a

Practice:

$2 + 2 \cdot 2 =$

$7 - 3(-5 + 2) =$

$-\left[(-3)^2 - (4-9)^2\right] =$

$\frac{-3}{4} \div \frac{5}{-6} =$

$\left(\frac{-2}{3}\right)^3 + \frac{2}{3} =$

$\left|-5 - 16 \div 2\right| =$

$-4\left|2 - \sqrt{0.25}\right| =$

$\frac{-3^2 - (-2)^3}{\sqrt{16} - 3|4-5|} =$

$11 - 2\left|8 \cdot (-2) \div (-2)^2\right| =$

<u>*Remember!*</u>

$-\frac{a}{b} = \frac{-a}{b} = \frac{a}{-b}$ and $\frac{-a}{-b} = \frac{a}{b}$, also $\sqrt{negative} = DNE$, so $\sqrt{x^2} = |x|$

Evaluate Expressions:

Example 3: Given that $a = -3,\ b = 4,$ and $c = -\frac{1}{2}$, evaluate the following:

a) $2a \div 3b$

b) $\frac{\sqrt{a+b} - 3a^2}{2c}$

Properties of Real Numbers:

Commutative Property $a + b = b + a$ and $ab = ba$

Associative Property $a + (b + c) = (a + b) + c$ and $a(bc) = (ab)c$

Inverse Property $a + (-a) = 0$ and $\frac{1}{a} \cdot a = 1 \quad if\ a \neq 0$

Identity Property $a + 0 = 0 + a = a$ and $a \cdot 1 = 1 \cdot a = a$

Multiplication by 0 $a \cdot 0 = 0$

Distributive Property $a(b \pm c) = ab \pm ac = (b \pm c)a$

Example 4:

$2x(3 + y) = 2x(3) + 2x(y) = 6x + 2xy$ but $2x(3)(y) \neq 2x(3) \cdot 2x(y)$

term - a product of numbers and variables, including variable expressions

ex. $2x$, πr^2, $-\frac{2}{3}x(x - 1)$, $-x^2y$, 1

like terms - terms with the same variables raised to the same powers

ex. $2m$, $-m$, $\frac{m}{2}$ but not $2x^2y$, $-xy^2$

Like terms can be **combined** by adding their **numerical coefficients.**

ex. $2 + \frac{x}{2} + 5x^2 - x + 1 - 3x^2 =$

Practice:

4. Simplify.

a) $-(-3x + 6y) - 2(x - 3y) =$

b) $3x(2y) - 5x(y - 2) - 7 =$