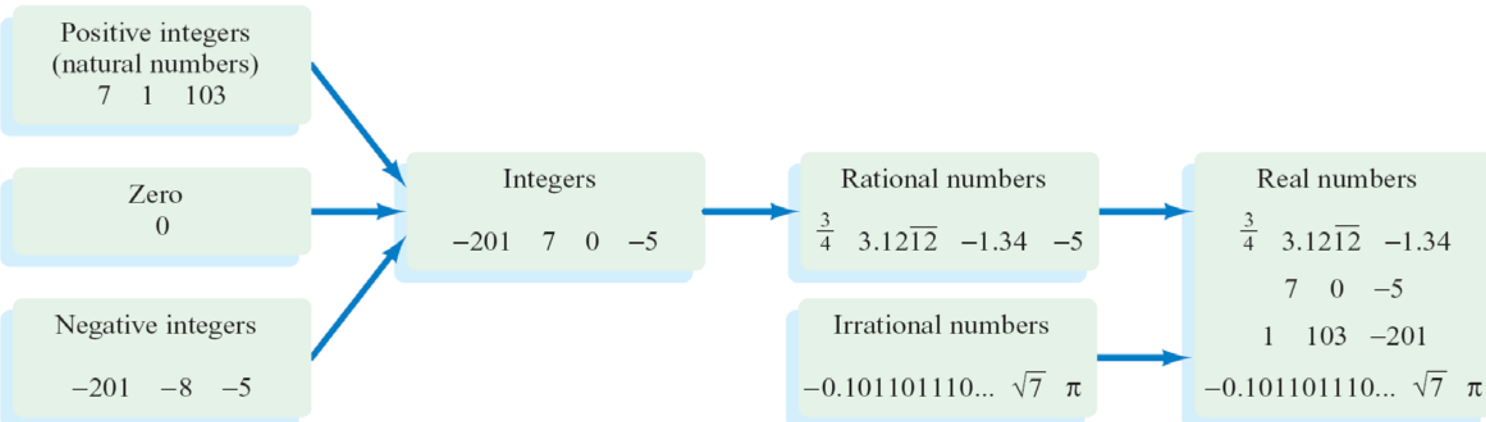
# P.1-P.2 Review of the Real Number System, Interval Notation and Rational Exponents

Human beings share the desire to organize and classify.

*Example:* Astronomers classify stars into groups called constellations.

In mathematics it is useful to place numbers with similar characteristics into **sets.** Generally, a **set** is a collection of objects, called **elements** (members of the set).




**set –** a collection of objects, called **elements,** having a common property

*Notation:*

**roster notation** – the elements of a set are listed in braces { } and separated by commas

ex. $\{1,2,3\}$ - **finite** set

$\mathbb{N} = \{1,2,3,\ldots\}$ - **natural** or **counting** numbers (**infinite** set)

$\mathbb{W} = \{0,1,2,3,\ldots\}$ - **whole** numbers

$\mathbb{Z} = \{\ldots,-2,-1,0,1,2,\ldots\} = \{0,\pm1,\pm2,\ldots\}$- **integers**

$\in$ **- is an element of,** ex. $5 \in \mathbb{N}$

$\notin$ **- is not an element of,** ex. $-1 \notin \mathbb{N}$

$\emptyset$ or { }- an **empty** (or **null**) set

$\mathbb{R}$ **- real** numbers (numbers that correspond to any point on a number line – those are called **coordinates**)

**set-builder notation** – the elements are determined by listing a property (or properties) that they need to satisfy

*Examples:*

$\mathbb{Q} = \left\{ \frac{p}{q} \,\middle|\, p, q \in \mathbb{Z},\ q \neq 0 \right\}$ - **rational** numbers (any fraction, finite decimal, or infinite but repeating decimal)

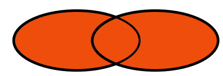
$\mathbb{IQ} = \{\boldsymbol{x} | \boldsymbol{x} \notin \mathbb{Q}\}$ - **irrational** numbers (any infinite but non-repeating decimal, root of a **prime** (divisible only by 1 or itself) number, constant $\boldsymbol{\pi}$ or $\boldsymbol{e}$)

$\mathbb{Z}_{-} = \{\boldsymbol{x} |\ \boldsymbol{x} \in \mathbb{Z}\ \ and\ \ x < 0\ \} = \{\ x \in \mathbb{Z}\ |\ x < 0\}$ - negative integers

$2\mathbb{Z} = \{\boldsymbol{n} \in \mathbb{Z} |\ \ \boldsymbol{n} = \boldsymbol{2k},\ \ \boldsymbol{k} \in \mathbb{Z}\}$ - even integers

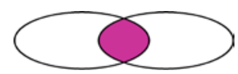
$\subset$ or $\subseteq$ - **inclusion** of sets;
ex. $\mathbb{N} \subset \mathbb{Z}$ ($\mathbb{N}$ is a **subset** of $\mathbb{Z}$); $\mathbb{N} \subseteq \mathbb{N}$ ($\mathbb{N}$ is an **improper subset** of $\mathbb{N}$)

$\cup$ - **union** of sets; $\boldsymbol{A} \cup \boldsymbol{B} = \{\boldsymbol{x} |\ \boldsymbol{x} \in \boldsymbol{A}\ \boldsymbol{or}\ \boldsymbol{x} \in \boldsymbol{B}\}$;



ex. $\{x |\ -2 < x < 3\} \cup \{x |\ 2 \le x \le 5\} =$

$\cap$ - **intersection** of sets; $A \cap \boldsymbol{B} = \{\boldsymbol{x} |\ \boldsymbol{x} \in \boldsymbol{A}\ \boldsymbol{and}\ \boldsymbol{x} \in \boldsymbol{B}\}$



ex. $\{x |\ -2 < x < 3\} \cap \{x |\ 2 \le x \le 5\} =$

**interval notation –** the way of representing any subset of real numbers corresponding to a segment or a half-line on a number line;

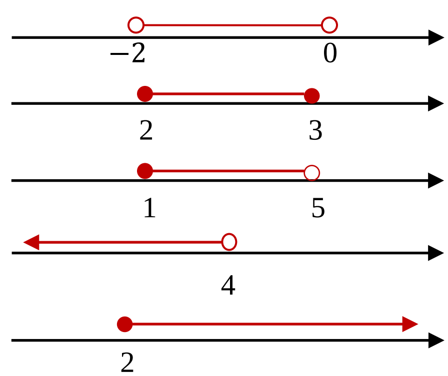
ex. $\{\ x\ |-2 < x < 0\} = (-2,0)$

$\{\ x\ |2 \le x \le 3\} = [2,3]$

$\{\ x\ |1 \le x < 5\} = [1,5)$

$\{\ x\ |x < 4\} = (-\infty, 4)$

$\{\ x\ |x \ge 2\} = [2, \infty)$

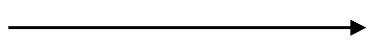



*Example 1:* Graph each set and simplify it, if possible.

A) $(-4,2) \cap [-1,7]$

B) $(-\infty, 3] \cap [3,5]$

C) $(-\infty, 2) \cup [3, \infty)$



D) $(2, \infty) \cup [1, \infty)$

**absolute value –** a distance of a number from zero; $|a| = \begin{cases} a & if\ a \geq 0 \\ -a & if\ a < 0 \end{cases}$

so **absolute value** of a number is always **positive!**

$|\boldsymbol{a} - \boldsymbol{b}|$ - **distance** between $\boldsymbol{a}$ and $\boldsymbol{b}$

*Example 2:* Translate into mathematical statement.

A) The distance between $\boldsymbol{x}$ and $-3$ is less than 5.

B) The distance between $\boldsymbol{x}$ and $\boldsymbol{y}$ is between 1 and 2.

*Example 3:* Simplify $|x-2| + |x+1|$ given that $-1 \leq x \leq 1$.

***Recall the Exponential Rules:***

$a^m \cdot a^n = a^{m+n}$

$\dfrac{a^m}{a^n} = a^{m-n}$

$(a^m)^n = a^{m \cdot n}$

$(ab)^n = a^n b^n$

$\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$

$\begin{cases} a^0 = 1, & if\ \ a \neq 0 \\ 0^0 & is\ undefined \end{cases}$

$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}, \quad if\ \ a \neq 0$

so $\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n, \quad if\ \ a, b \neq 0$

and $\dfrac{a^{-n}}{b^{-m}} = \dfrac{b^m}{a^n}, \quad if\ \ a, b \neq 0$

$x^{\frac{1}{n}} = \sqrt[n]{x}$

$x^{\frac{m}{n}} = \sqrt[n]{x^m}$

$\sqrt[n]{\boldsymbol{x^n}} = \begin{cases} |\boldsymbol{x}|, & if\ \boldsymbol{n}\ is\ \boldsymbol{even} \\ \boldsymbol{x}, & if\ \boldsymbol{n}\ is\ \boldsymbol{odd} \end{cases}$

*Example 4:* Simplify.

A) $-32^{-\frac{3}{5}}$

B) $\left(\dfrac{-3x^4y^{-2}}{15x^{-6}y^5}\right)^{-3}$

C) $\dfrac{a^{\frac{1}{3}}}{a^{-\frac{1}{4}} \cdot a^{\frac{2}{3}}}$

D) $\sqrt[4]{3^5 a^{14} b^7}$

E) $\sqrt{3a} + 2\sqrt{27a^5}$

F) $\left(3\sqrt{x} + 5\right)^2$

*Example 5:* Rationalize the denominator.

A) $\sqrt{\dfrac{3}{20}}$

B) $\dfrac{2}{3\sqrt{5} - 2\sqrt{3}}$

*Example 6:* Rationalize the numerator.

$$\frac{4 - \sqrt{16 - h}}{h}$$