P.4-P.5 Review of Factoring and Operations on Rational Expressions

Factoring Strategies:

1. **Greatest Common Factor**

$-12x^4y + 16x^3y^2 - 8xy^3 =$

$2x(x-1) + 3y(x-1) =$

$(3x+2)(x+5) - (2x-1)(3x+2) =$

$2(x+5)^{-3} - 6(x+5)^{-5} - 4(x+5)^{-4} =$

2. **Factoring by Grouping**

$6xy - 4y + 15x - 10 =$

$5x^2y - x^3y - 10 + 2x =$

3. **Factoring Trinomials with the Leading Coefficient = 1** (guessing method)

$$x^2 + \underbrace{\boldsymbol{b}}_{(\boldsymbol{p}+\boldsymbol{q})} x + \underbrace{\boldsymbol{c}}_{\boldsymbol{pq}} = (x+p)(x+q)$$

$3x^2y^2 + 15xy - 72 =$

$x^4 + 13x^2y^3 + 42y^6 =$

$x^6(x-3) - 2x^3(x-3) + x - 3 =$

4. **Factoring Trinomials with the Leading Coefficient ≠ 1**

$$\underbrace{\boldsymbol{a}}_{\boldsymbol{mn}} x^2 + \underbrace{\boldsymbol{b}}_{(\boldsymbol{mq}+\boldsymbol{np})} x + \underbrace{\boldsymbol{c}}_{\boldsymbol{pq}} = (\boldsymbol{m}x + p)(\boldsymbol{n}x + q)$$

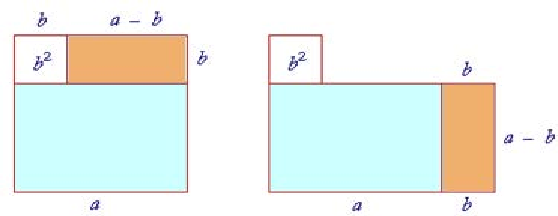
$10x^2 - 3x - 1 =$

$2x^2y^2 + 3xy - 9 =$

$20p^4 - 23p^2q^2 + 6q^4 =$

5. **Special Factoring**

Difference of Squares: $\boldsymbol{a^2 - b^2 = (a+b)(a-b)}$



$25a^2 - 64b^2 =$

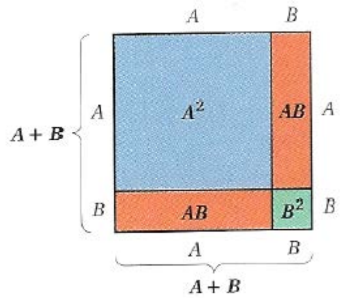
$20x^4y - 125y^3 =$

$(x+y)^2 - 100 =$

$49y^2 - (x-3)^2 =$

<u>Attention:</u> **Sum of squares is NOT factorable**. Ex. $x^2 + 9$ is **prime**.

Perfect Square: $\boldsymbol{a^2 + 2ab + b^2 = (a+b)^2}$
$\boldsymbol{a^2 - 2ab + b^2 = (a-b)^2}$



$16a^2 + 8ab + b^2 =$

$81n^2 + 144nm + 64m^2 =$

$-9x^3y^2 + 30x^2y - 25x =$

$(x-y)^2 - 2(x-y) + 1 =$

$x^2 + 16x + 64 - 16y^2 =$

$25 - a^2 + 6ab - 9b^2 =$

Sum or Difference of Cubes: $\boldsymbol{a^3+b^3=(a+b)(a^2-ab+b^2)}$
$\boldsymbol{a^3-b^3=(a-b)(a^2+ab+b^2)}$

$x^3+8=$

$64a^3-27b^3=$

$-x^3-y^6=$

$(x+1)^3+125=$

Example 1: Factor completely.

a) $y^5+y^4-y-1=$

b) $5x^3-5x^2y-5xy^2+5y^3=$

c) $4a^2-4a+1-b^2+6b-9=$

d) $4(a+b)^2-19(a+b)-5=$ (*hint:* let $x=a+b$, and factor the related polynomial $4x^2-19x-5$)

e) $8(x-3)^2-64(x-3)+128=$

To **simplify**, **multiply** or **divide rational expressions**, factor completely each numerator and denominator; then reduce:

$$\frac{a+4}{a^2}\cdot\frac{a^2+4a}{a^2-16}\div\frac{a^2+8a+15}{a^2+a-20}=$$

To **add** or **subtract rational expressions**, rewrite them using LCD:

$$\frac{2}{x+6} - \frac{x+2}{x^2-36} + \frac{5}{x-6} =$$

$$\frac{3x}{x^2 - 7x + 10} - \frac{3}{x^2 - 8x + 15} =$$

To **simplify complex fractions:**

- change the numerator and denominator into single fractions and then change the main division into multiplication by reciprocal, or
- multiply the numerator and denominator by the LCD of all the fractions.

$$\frac{\frac{x+2}{x} - \frac{1}{x+2}}{\frac{5}{x} + \frac{x}{x+2}} =$$

or

$$\frac{\frac{x+2}{x} - \frac{1}{x+2}}{\frac{5}{x} + \frac{x}{x+2}} =$$

$$1 + \frac{1}{1+\frac{1}{x}} =$$

$$\frac{x^{-1} - y^{-1}}{\frac{x-y}{xy}} =$$