2.3, 2.4 Applications of Linear Equations

To solve an applied problem, we need to translate words into algebraic expressions.

Here are the most commonly used key words suggesting a particular operation:

ADDITION (+)	**SUBTRACTION (−)**	**MULTIPLICATION (·)**	**DIVISION (÷)**
sum	**difference**	**product**	**quotient**
plus	minus	multiply	divide
add	**subtract from**	times	ratio
total	**less than**	**of**	**out of**
more than	less	half of	per
increase by	decrease by	half as much as	shared
together	diminished	twice	cut into
perimeter		triple	
		area	

Example 1: Translate from the word description into an algebraic expression or equation.

phrase or sentence:	**expression or equation:**
the **sum** of double a number **and** one	
the difference of squares of two numbers	
Half of a number, decreased by 3, is 18.	
The quotient of a number and 4, plus the number, equals to 10.	
The quotient of a number and 4 plus the number, equals to 10.	
The area of a square with a side ***a*** is 20.	
A number subtracted from 5 is 1 less than the number.	
perfect square of the sum of two numbers	
4% of the product of two consecutive even numbers	

General Guidelines and Hints for Solving Word Problems:

- **read** the problem twice
  - first time focus on the general setting; try to recognize what type of problem it is: motion, investment, proportion, geometry, age, mixture or solution, work, number, etc.;
  - second time focus on information that is given and what needs to be found;
- draw appropriate tables or diagrams to picture out the situation and **organize your data** in a useful way; list relevant formulas;
- **assign a variable**(s) for the unknown (this is usually what the problem asks for);
  - write a "let" statement or label your diagram to describe clearly what your variable stands for and what are the units;
  - express other unknown values in terms of your variable(s);
- **write an equation**(s) using the unknown expressions by following a relevant formula(s), for example: the formula used in the table, geometry, or any other common sense pattern;
- **solve** the equation(s);
- **check** if the solution is reasonable; check if your denominations have sense;
- **give the formal answer**.

Commonly used patterns, formulas, and tables:

Numerical Problems

- watch for relations between numerical quantities,
- list **consecutive** numbers as: $\boldsymbol{n,\ n+1,\ n+2,\ \dots}$
- list **consecutive even or odd** numbers as: $\boldsymbol{n,\ n+2,\ n+4,\ \dots}$

Example 1: Find two consecutive even numbers such that three times the first plus twice the second is 254.

Solution: Let the two consecutive even numbers be: $n,\ n+2$

Answer:

% Problems

use the fact $\mathbf{1 = 100\%}$ and the equation $\boxed{\dfrac{\textit{is a part}}{\textit{of a whole}} = \dfrac{\%}{100}}$

Example 2: The average price of a 1bd apartment in BC rose from \$115000 in 2009 to \$121000 in 2010. What percent increase was this, to the nearest tenth of a percent?

Solution: First, find the increase.

Then, let ***p*** be the percent increase.

Answer: The average price of a 1bd apartment in BC increased by .... between 2009 and 2010.

Number-Value Problems

use the table:

	item A	item B	total
# of items			
value of items			

Example 3: For a bill totalling \$2.55, a cashier received 15 coins consisting of dimes and quarters. How many of each denomination of coins did the cashier receive?

Solution: Let ***d*** represents the number of **dimes**, then
.... represents the number of **quarters**

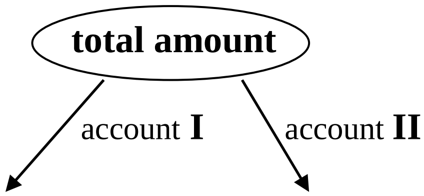
	dimes	quarters	total
# of coins			
value of coins			

Answer: The cashier received .... dimes and .... quarters.

Investment Problems

use the formula $\boldsymbol{I = Prt}$ for simple interest

and the diagram



interest I + **interest II** = **total interest**

or

interest I = **interest II**

Example 4: Peter invested some money at 3.5% simple interest, and \$5000 more than three times the amount at 4%. He earned \$1440 in annual interest. How much did he invest at each rate?

Solution: Let $\boldsymbol{x}$ be the amount invested at 3.5%, then
.... is the amount invested at 4%.

Answer: He invested .... at 3.5% and .... at 4%.

Geometry Problems

useful facts to remember:

sum of $\angle' s$ in a $\Delta = 180°$

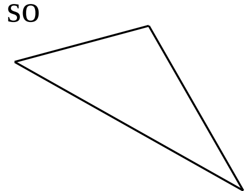
perimeter of a:
- rectangle = $2a + 2b$
- circle = $2\pi r$

Pythagorean Theorem: $a^2 + b^2 = c^2$

area of a
- triangle = $\frac{1}{2} bh$
- rectangle or parallelogram = bh
- circle = πr^2
- trapezoid = $\frac{1}{2}(a + b)h$

Example 5: The second angle of a triangle is three times as large as the first angle. The measure of the third angle is 10° less than that of the first angle. How large are the angles?

Solution: Notice that all the angles in the problem refer to the first angle, so let $\boldsymbol{x}$ denotes the **first angle**.



Answer: The three angles are:

Mixture-Solution Problems

use the table:

	% ·	volume =	content
Category I			
Category II			
mix/solution			

Example 6: An automobile radiator has a capacity of 16 litres. How much pure antifreeze must be added to a mixture of water and antifreeze that is 18% antifreeze to make a mixture of 20% antifreeze that can be used to fill the radiator?

Solution: Let ***x*** be the volume of the pure antifreeze, in litres, then .... represents the volume of the 18% antifreeze, in litres.

	% ·	volume =	antifreeze
18% antifreeze			
pure antifreeze			
mix/solution			

Answer: .... litres of pure antifreeze needs to be added to approximately .... litres of 18% antifreeze.

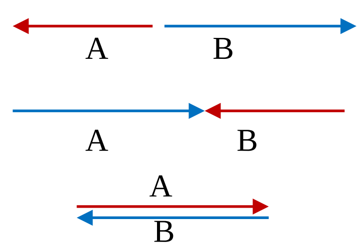
Motion Problems

use the formula $\boldsymbol{R \cdot T = D}$ and the table:

	rate ·	time =	distance
situation I			
situation II			
total			

Notice! Sometimes, the last row is not needed.

possible situations:



Example 7: A Coast-Guard patrol boat travels 4 hr on a trip downstream with a 6-mph current. The return trip against the same current takes 5 hr. Find the speed of the boat in still water.

Solution: Let ***r*** be the speed of the boat in still water, then
.... represents the downstream speed of the boat, and
.... represents the upstream speed of the boat.

	rate ·	**time** =	**distance**
downstream			
upstream			

Answer: The speed of the boat in still water is ....