## L. 3 Applications of Linear Equations

To solve an applied problem, we need to translate words into algebraic expressions.
Here are the most commonly used key words suggesting a particular operation:

| ADDITION (+) | SUBTRACTION ( - ) | MULTIPLICATION ( $\cdot$ ) | DIVISION ( $\div$ ) |
| :--- | :--- | :--- | :--- |
| sum | difference | product | quotient |
| plus | minus | multiply | divide |
| add | subtract from | times | ratio |
| total | less than | of | per |
| more than | less | half of | shared |
| increase by | decrease by | half as much as | cut into |
| longer | diminished | twice |  |
| together | shorter | triple | area |
| perimeter |  |  |  |

Example 1: Translate each word description into an algebraic expression or equation.
phrase or sentence:
the sum of double a number and one
the difference of squares of two numbers
Half of a number, decreased by 3 , is 18 .
The quotient of a number and 4 , plus the number, equals to 10 .

The quotient of a number and 4 plus the number equals to 10 .

The area of a square with a side $\boldsymbol{a}$ is 20 .
A number subtracted from 5 is 1 less than the number.
perfect square of the sum of two numbers
$4 \%$ of the product of two consecutive even numbers

## General Guidelines and Hints for Solving Word Problems:

- read the problem twice
$>$ first time focus on the general setting; try to recognize what type of problem it is: motion, investment, proportion, geometry, age, mixture or solution, work, number, etc.;
$>$ second time focus on information that is given and what needs to be found;
- draw appropriate tables or diagrams to picture out the situation and organize your data in a useful way; list relevant formulas;
- assign a variable(s) for the unknown (this is usually what the problem asks for);
> write a "let" statement or label your diagram to describe clearly what your variable stands for and what are the units;
> express other unknown values in terms of your variable(s);
- write an equation(s) using the unknown expressions by following a relevant formula(s), for example: the formula used in the table, geometry, or any other common sense pattern;
- solve the equation(s);
- check if the solution is reasonable; check if your denominations have sense;
- give the formal answer.


## Commonly used patterns, formulas, and tables:

## Numerical Problems

> watch for relations between numerical quantities,
$>$ list consecutive numbers as: $\quad \boldsymbol{n}, \boldsymbol{n}+\mathbf{1}, \boldsymbol{n}+2, \ldots$
$>$ list consecutive even or odd numbers as: $n, n+2, n+4, \ldots$
Example 1: Find two consecutive even numbers such that three times the first plus twice the second is 254 .

Solution: Let the two consecutive even numbers be: $n, n+2$

## \% Problems

use the fact $\quad \mathbf{1}=\mathbf{1 0 0} \% \quad$ and the equation $\quad \frac{\text { is a part }}{\text { of a } \text { whole }}=\frac{\%}{\mathbf{1 0 0}}$
Example 2: The average price of a 1bd apartment in BC rose from $\$ 115000$ in 2009 to $\$ 121000$ in 2010. What percent increase was this, to the nearest tenth of a percent?

Solution: First, find the increase.

Then, let $\boldsymbol{p}$ be the percent increase.

Answer: The average price of a 1bd apartment in BC increased by $\qquad$ between 2009 and 2010.

## Number-Value Problems

use the table:

|  | item A | item B | total |
| :--- | :--- | :--- | :--- |
| \# of items |  |  |  |
| value of items |  |  |  |

Example 3: For a bill totalling $\$ 2.55$, a cashier received 15 coins consisting of dimes and quarters. How many of each denomination of coins did the cashier receive?

Solution: Let $\boldsymbol{d}$ represents the number of dimes, then
.......... represents the number of quarters

|  | dimes | quarters | total |
| :--- | :--- | :--- | :--- |
| \# of coins |  |  |  |
| value of coins |  |  |  |

Answer: The cashier received $\qquad$ dimes and $\qquad$ quarters.

## Investment Problems

use the formula $\boldsymbol{I}=\boldsymbol{P r t}$ for simple interest
and the diagram


Example 4: Peter invested some money at $3.5 \%$ simple interest, and $\$ 5000$ more than three times the amount at $4 \%$. He earned $\$ 1440$ in annual interest. How much did he invest at each rate?
Solution: Let $\boldsymbol{x}$ be the amount invested at $3.5 \%$, then
$\qquad$ is the amount invested at $4 \%$.

Answer: He invested $\qquad$ at $3.5 \%$ and $\qquad$ at $4 \%$.

## Geometry Problems

useful facts to remember:
sum of $\angle ' s$ in a $\Delta=180^{\circ}$
area of a
perimeter of a:

- triangle $=\frac{1}{2} b h$
- rectangle $=2 a+2 b$
- rectangle or parallelogram = bh
- circle $=2 \pi r$
- circle $=\pi r^{2}$

Pythagorean Theorem: $a^{2}+b^{2}=c^{2}$

- $\quad$ trapezoid $=\frac{1}{2}(a+b) h$

Example 5: The second angle of a triangle is three times as large as the first angle. The measure of the third angle is $10^{\circ}$ less than that of the first angle. How large are the angles?

Solution: Notice that all the angles in the problem refer to the first angle, so let $\boldsymbol{x}$ denotes the first angle.

Answer: The three angles are:


## Mixture-Solution Problems

use the table:

|  | \% | $\cdot$ | volume $=$ |
| :--- | :--- | :--- | :--- |
| content |  |  |  |
| Category I |  |  |  |
| Category II |  |  |  |
| mix/solution |  |  |  |

Example 6: An automobile radiator has a capacity of 16 litres. How much pure antifreeze must be added to a mixture of water and antifreeze that is $18 \%$ antifreeze to make a mixture of $20 \%$ antifreeze that can be used to fill the radiator?

Solution: Let $\boldsymbol{x}$ be the volume of the pure antifreeze, in litres, then
$\qquad$ represents the volume of the $18 \%$ antifreeze, in litres.

|  | $\%$ | volume $=$ | antifreeze |
| :--- | :--- | :--- | :--- |
| $18 \%$ antifreeze |  |  |  |
| pure antifreeze |  |  |  |
| mix/solution |  |  |  |



Answer: $\qquad$ litres of pure antifreeze needs to be added to approximately litres of $18 \%$ antifreeze.

## Motion Problems

use the formula $\boldsymbol{R} \cdot \boldsymbol{T}=\boldsymbol{D}$ and the table:

|  | rate $\cdot$ | time $=$ | distance |
| :--- | :--- | :--- | :--- |
| motion I |  |  |  |
| motion II |  |  |  |
| total |  |  |  |

Notice! Sometimes, the last row is not needed.

Example 7: A Coast-Guard patrol boat travels 4 hr on a trip downstream with a 6-mph current. The return trip against the same current takes 5 hr . Find the speed of the boat in still water.

Solution: Let $\boldsymbol{r}$ be the speed of the boat in still water, then
.................. represents the downstream speed of the boat, and .................. represents the upstream speed of the boat.

|  | rate $\cdot$ | time $=$ | distance |
| :--- | :--- | :--- | :--- |
| downstream |  |  |  |
| upstream |  |  |  |



Answer: The speed of the boat in still water is $\qquad$

