L.3 Applications of Linear Equations

To solve an applied problem, we need to translate words into algebraic expressions.

ADDITION (+)	SUBTRACTION (-)	MULTIPLICATION (\cdot)	DIVISION (÷)
sum	difference	product	quotient
plus	minus	multiply	divide
add	subtract from	times	ratio
total	less than	of	out of
more than	less	half of	per
increase by	decrease by	half as much as	shared
longer	diminished	twice	cut into
together	shorter	triple	
perimeter		area	

Here are the most commonly used key words suggesting a particular operation:

Example 1: Translate each word description into an algebraic expression or equation.

phrase or sentence:

the **sum** of double a number **and** one

the difference of squares of two numbers

Half of a number, decreased by 3, is 18.

The quotient of a number and 4, plus the number, equals to 10.

The quotient of a number and 4 plus the number equals to 10.

The area of a square with a side *a* is 20.

A number subtracted from 5 is 1 less than the number.

perfect square of the sum of two numbers

4% of the product of two consecutive even numbers

expression or equation:

Lecture 2.3, 2.4

General Guidelines and Hints for Solving Word Problems:

- **read** the problem twice
 - first time focus on the general setting; try to recognize what type of problem it is: motion, investment, proportion, geometry, age, mixture or solution, work, number, etc.;
 - second time focus on information that is given and what needs to be found;
- draw appropriate tables or diagrams to picture out the situation and **organize your data** in a useful way; list relevant formulas;
- **assign a variable**(s) for the unknown (this is usually what the problem asks for);
 - write a "let" statement or label your diagram to describe clearly what your variable stands for and what are the units;
 - express other unknown values in terms of your variable(s);
- write an equation(s) using the unknown expressions by following a relevant formula(s), for example: the formula used in the table, geometry, or any other common sense pattern;
- **solve** the equation(s);
- **check** if the solution is reasonable; check if your denominations have sense;
- give the formal answer.

Commonly used patterns, formulas, and tables:

Numerical Problems

- ➤ watch for relations between numerical quantities,
- > list consecutive numbers as: n, n + 1, n + 2, ...
- > list consecutive even or odd numbers as: n, n + 2, n + 4, ...

Example 1: Find two consecutive even numbers such that three times the first plus twice the second is 254.

Solution: Let the two consecutive even numbers be: n, n + 2

Answer:

Math 085 (Anne	a K.)	<i>Lecture 2.3, 2.4</i>					
% Problem	ıs						
use the fact	1 = 100%	and the equation	$\frac{is \ a \ part}{of \ a \ whole} = \frac{\%}{100}$				

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Example 2: The average price of a 1bd apartment in BC rose from \$115000 in 2009 to \$121000 in 2010. What percent increase was this, to the nearest tenth of a percent?

Solution: First, find the increase.

Then, let *p* be the percent increase.

Answer: The average price of a 1bd apartment in BC increased by between 2009 and 2010.

Number-Value Problems

use the table:

	item A	item B	total
# of items			
value of items			

Example 3: For a bill totalling \$2.55, a cashier received 15 coins consisting of dimes and quarters. How many of each denomination of coins did the cashier receive?

Solution: Let *d* represents the number of **dimes**, then represents the number of **quarters**

	dimes	quarters	total
# of coins			
value of coins			



Example 4: Peter invested some money at 3.5% simple interest, and \$5000 more than three times the amount at 4%. He earned \$1440 in annual interest. How much did he invest at each rate?

Solution: Let x be the amount invested at 3.5%, then is the amount invested at 4%.

Answer: He invested at 3.5% and at 4%.

Geometry Problems

useful facts to remember:

sum of $\angle s$ in a $\triangle = 180^{\circ}$ perimeter of a: - rectangle = 2a + 2b- circle = $2\pi r$ Pythagorean Theorem: $a^2 + b^2 = c^2$ area of a - triangle = $\frac{1}{2}bh$ - rectangle or parallelogram = bh- circle = πr^2 - trapezoid = $\frac{1}{2}(a + b)h$

Example 5: The second angle of a triangle is three times as large as the first angle. The measure of the third angle is 10° less than that of the first angle. How large are the angles?

Solution: Notice that all the angles in the problem refer to the first angle, so let x denotes the **first angle**.

Answer: The three angles are:

Mixture-Solution Problems

use the table:

	%.	volume =	content
Category I			
Category II			
mix/solution			

Example 6: An automobile radiator has a capacity of 16 litres. How much pure antifreeze must be added to a mixture of water and antifreeze that is 18% antifreeze to make a mixture of 20% antifreeze that can be used to fill the radiator?

	%	•	volume =	antifreeze
18% antifreeze				
pure antifreeze				
mix/solution				

Answer: litres of pure antifreeze needs to be added to approximately litres of 18% antifreeze.

Motion Problems

use the formula $\mathbf{R} \cdot \mathbf{T} = \mathbf{D}$ and the table:

	rate	time	=	distance
motion I				
motion II				
total				

Notice! Sometimes, the last row is not needed.

possible situations:



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Example 7: A Coast-Guard patrol boat travels 4 hr on a trip downstream with a 6-mph current. The return trip against the same current takes 5 hr. Find the speed of the boat in still water.

Solution: Let **r** be the speed of the boat in still water, then represents the downstream speed of the boat, and represents the upstream speed of the boat.

	rate	•	time	=	distance
downstream					
upstream					



Answer: The speed of the boat in still water is