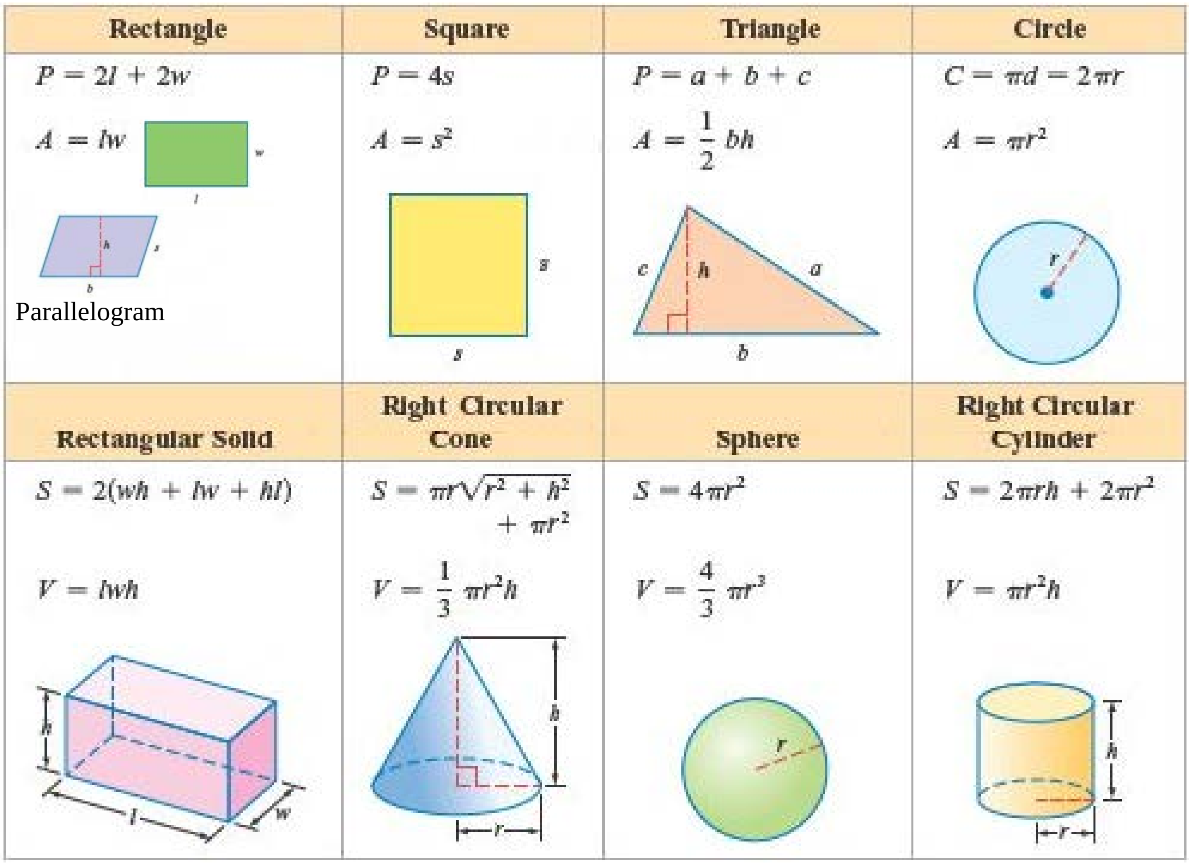
1.2 <u>Solving Formulas and Applied Problems</u>

formula – a general rule describing relationship between various quantities (variables)

Some formulas from geometry:



When solving formulas for a given variable, remember to

- **highlight** the variable of interest and **treat other variables as numbers**
  ex. $2L + 2W = P$ solve for L

- think about **undoing** (reversing) **operations** to reach the given variable;

  here are the pairs of reversing operations:
  $+$ $-$
  $\cdot$ $\div$
  $(*)^2$ $\sqrt{*}$

  ex. $a_n = a_1 + (n-1)d$ solve for n

- keep the **variable in the numerator**
  ex. $S = \frac{a_1}{1-r}$ solve for r

- keep the given **variable in one place**
  ex. $S - Sdt = P$ solve for S

Recall General Guidelines and Hints for Solving Applied Problems:

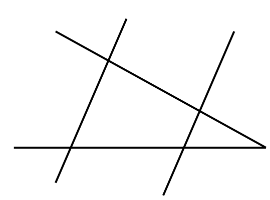
- **read** the problem twice
- recognize the type of the problem and draw appropriate tables or diagrams to **organize your data** in a useful way; list relevant formulas;
- **assign variables** for the unknown quantities, use meaningful letters;
- **write equations** by following a relevant formula(s) or a common sense pattern;
- **solve** the system of equations;
- **check** if the solution is reasonable;
- **give the formal answer**.

Commonly used tables, patterns, and formulas:

Geometry Problems

Example 1: A building casts a shadow 20 feet long. A person 6 feet tall is walking directly away from the building toward the edge of the building's shadow. How tall is the building if the person is 15 ft from the building when the shadow of the person and the shadow of the building line up?

Solution: **Similar Triangles (Thales) Theorem:**
Ratios of corresponding sides of **similar** (having the same angles) triangles are equal.



Let ***x*** be the height of the building.

Answer: The building is .... ft tall.

Business Problems (Profit = Revenue – Cost)

Example 2: It costs a manufacturer \$8.95 to produce sunglasses that sell for \$29.99. How many pairs of sunglasses must be sold to make a profit of \$17,884?

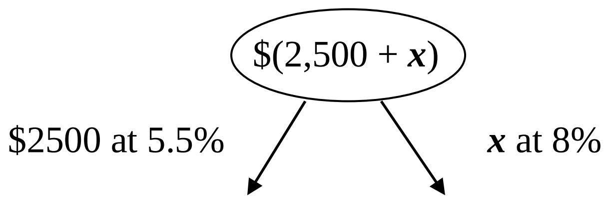
Solution: Let ***n*** be the number of sunglasses sold.

Answer: The manufacturer must sell .... sunglasses.

Investment Problems ($\boldsymbol{I = Prt}$)

Example 3: An investment of \$2500 is made at an annual simple interest rate of 5.5%. How much additional money must be invested at an annual simple interest rate of 8%, so that the total interest earned is 7% of the total investment?

Solution: Let ***x*** = the money invested at 8%.



interest:

Answer: It must be invested .... at 8% .

Motion Problems ($\boldsymbol{R \cdot T = D}$)

Example 4: A car traveling at 80 kph is passed by a second car going in the same direction at a constant speed. After 30 seconds, the two cars are 500 metres apart. Find the speed of the second car.

Solution: Let ***v*** be the speed of the second car.

Rate ·	**Time =**	**Distance**

Answer: The speed of the second car is..................

Mixture (Solution) Problems ($\% \cdot \text{volume} = \text{content}$)

Example 5: A radiator contains 6 litres of a 25% antifreeze solution. How much should be drained and replaced with pure antifreeze to produce a 33% antifreeze solution?

Solution: Let ***x*** be the volume of pure antifreeze.

	% ·	**volume =**	**content**
25% solution			
pure antifreeze			
solution			

Answer: .... L of 25% solution must be replaced by pure antifreeze.

Work Problems (**Rate** of performing the job · **Time =** amount of **Job** done (usually 1))

Remember: In work problems we usually ***add rates***, but **not times!**

Example 6: David can paint a house in 12 hours. Bill can paint the same house in 9 hours. How long would it take them to paint the house together?

Solution: Let ***t*** be the time needed to paint the house together.

	R ·	**T =**	**job done**
David			1
Bill			1
together			1

Answer: It will take them .... hours, if working together.

Example 7:

A tank can be filled in 9 hr and drained in 11 hr. How long will it take to fill the tank if the drain is left open?

Solution: Let ***t*** be the time needed to fill the tank if the drain is left open.

	R ·	**T =**	**job done**
filling in			1
draining out			1
together			1

Answer: It will take .... to fill the tank.