1.3 Quadratic Equations and Their Applications

Quadratic equation – any equation that can be written in the form $\boldsymbol{ax^2 + bx + c = 0}$, where $a, b, c \in \mathbb{R}$, $a \neq 0$. This is called **standard form**.

Different ways of solving a quadratic equation:

Method:	*Examples:*
Factoring - use **Zero-Product Rule:** If $AB = 0,$ then $A = 0$ or $B = 0$	$x^2 - 4 = 0$ solution set = $3x^2 + 2x - 5 = 0$ solution set =
Square Root Property *Recall:* $\sqrt{x^2} = \lvert x \rvert$, $\sqrt{-1} = \boldsymbol{i}$ (**imaginary number**)	$x^2 - 4 = 0$ solution set = $(x-1)^2 = -4$ solution set =
Completing the Square - isolate the constant term - divide by the leading coefficient - complete the square by **adding half of the middle coefficient** to both sides of the equation, - and solve by square root property	$x^2 - 2x - 5 = 0$ $x^2 - 2x = 5$ $x^2 - 2x + 1 = 5 + 1$ $(x-1)^2 = 6$ solution set = $\{ 1 \pm \sqrt{6} \}$ $2x^2 + 3x - 1 = 0$ solution set =

Quadratic Formula $\boldsymbol{x}_{1,2}=\dfrac{-\boldsymbol{b}\pm\sqrt{\boldsymbol{b}^2-4\boldsymbol{ac}}}{2\boldsymbol{a}}$	$3x^2+2x-5=0$ coefficients: $\boldsymbol{a}=3, \boldsymbol{b}=2, \boldsymbol{c}=-5$ $x_{1,2}=\dfrac{-2\pm\sqrt{2^2-4\cdot 3(-5)}}{2\cdot 3}=$ solution set =

Let us develop the **Quadratic Formula** $\boldsymbol{x}=\frac{-\boldsymbol{b}\pm\sqrt{\boldsymbol{b}^2-4\boldsymbol{ac}}}{2\boldsymbol{a}}$ for solutions of the equation $\boldsymbol{ax^2+bx+c=0}$, by following the completing the square method:

Notice 1: The number of solutions of a quadratic equation depends on the expression $\boldsymbol{b^2-4ac}$, called the **discriminant** and often denoted $\boldsymbol{\Delta}$:
if $\boldsymbol{\Delta}>0$, then we have **two** distinct **real solutions**;
if $\boldsymbol{\Delta}=0$, then we have **one** double **real solution**;
if $\boldsymbol{\Delta}<0$, then we have two distinct nonreal (complex) solutions;

Quadratic Formula, in terms of the discriminant $\boldsymbol{\Delta}=\boldsymbol{b^2-4ac}$, takes the form:

$$\boldsymbol{x}=\frac{-\boldsymbol{b}\pm\sqrt{\boldsymbol{\Delta}}}{2\boldsymbol{a}}\ .$$

Notice 2: If the discriminant is a perfect square, then the equation can be solved by factoring; otherwise, we need to use the quadratic formula.

Example 1: Determine the number of real solutions, without solving the equation.

a) $3p^2 = -8p - 1$ b) $x^2 + x + 1 = 0$ c) $y^2 - \frac{2}{3}y + \frac{1}{9} = 0$

Example 2: Solve by using the **Quadratic Formula**.

a) $-2t(t + 2) = -3$ b) $3x^2 + 2x + 1 = 0$

Example 3: Solve by **completing the square**.

a) $x^2 + 3x - 1 = 0$ b) $4x^2 - 4x + 6 = 0$

Example 4: The demand for a certain product is given by $\boldsymbol{p = 26 - 0.01x}$, where $\boldsymbol{x}$ is the number of units sold per month and $\boldsymbol{p}$ is the price, in dollars, at which each item is sold. The monthly revenue is given by $\boldsymbol{R = xp}$. What number of items sold produces monthly revenue of $16,500?

Example 5: A veterinarian wishes to use 132 feet of fencing to enclose a rectangular region and subdivide it into two smaller regions, as shown. If the total enclosed area is 576 ft^2, find the dimensions of the enclosed region.