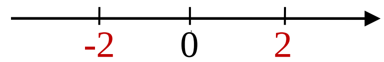
L.6 Absolute Value Equations and Inequalities

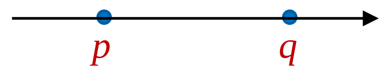
Remember: Absolute value represents "**distance from zero**", so $|x| = 2$ tells us that x is 2 steps from zero; therefore $x = 2$ or $x = -2$.



Generally, to solve an absolute value equation $|\boldsymbol{expression}| = \boldsymbol{k}$,
we must consider **two cases**

$$\boldsymbol{expression} = \boldsymbol{k} \quad \text{or} \quad \boldsymbol{expression} = -\boldsymbol{k}$$

and solve them separately.
The solution set usually consists of two numbers $\{p, q\}$.



Example 1: Solve.

a) $|3x + 2| = 14$

b) $2|x| - 1 = 3$ ***isolate** abs. value first!*

c) $\left|\frac{3x+2}{3}\right| = 5$

d) $|1 - x| = -2$ *abs. value can't be negative!*

To solve equations with two absolute values, follow the pattern:

$$|\boldsymbol{expr.A}| = |\boldsymbol{expr.B}|$$

$$\boldsymbol{expr.A} = \boldsymbol{expr.B} \quad \text{or} \quad \boldsymbol{expr.A} = -\boldsymbol{expr.B}$$

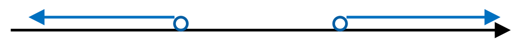
Example 2: Solve.

$$\left|\frac{x}{2} - 5\right| = \left|3 - \frac{x}{2}\right|$$

Generally, there are two types of absolute value inequalities:

$$|expression| < k$$
$$\Updownarrow$$
$$-k < expression < k$$

$$|expression| > k$$
$$expression < -k \quad \textbf{or} \quad expression > k$$



Example 3: Solve. Graph the solution on a number line and state it in interval notation.

a) $|-1 - 2x| < 5$

b) $\left|\frac{x-2}{3}\right| \geq 4$

c) $-|2x - 3| \geq -7$

d) $\left|\frac{1}{3}x + 7\right| + 5 > 6$

Watch these special cases:

e) $|5x + 2| < -8$

f) $-2|3x - 4| < 16$