

1.4 Polynomial, Rational, Radical Equations, and Their Applications

➤ **polynomial equations** (solve by factoring)

Example 1: Solve.

a) $x^3 - 2x^2 - 9x + 18 = 0$

b) $x^8 - 17x^4 + 16 = 0$

➤ **reducible to quadratic equations** (solve as the related quadratic equation)

Example 2: Solve.

a) $3x^{\frac{2}{3}} - 11x^{\frac{1}{3}} - 4 = 0$

b) $6x - 7\sqrt{x} - 20 = 0$

➤ **rational exponents equations** $\left((x^n)^{\frac{1}{n}} = \begin{cases} |x|, & \text{if } n \text{ is even} \\ x, & \text{if } n \text{ is odd} \end{cases} \right)$

Example 3: Solve.

a) $5x^{\frac{3}{4}} + 4 = 44$

b) $4x^{\frac{4}{5}} - 27 = 37$

➤ **rational equations** (solve by multiplying by **LCD**; check solutions against the **domain**)

Example 4: Solve.

a) $2x + \frac{3}{x-1} = \frac{-7x+10}{x-1}$

b) $\frac{4-3x}{2x+1} + \frac{3x+2}{x+2} = \frac{4x-5}{2x+1}$

➤ **radical equations** (isolate single radical; square both sides; check against extraneous solutions)

Example 5: Solve.

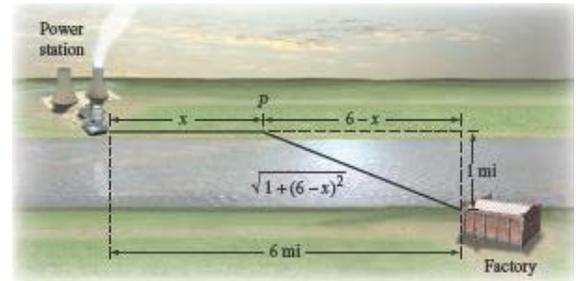
a) $\sqrt{5x-1} + 5 = x$

b) $\sqrt{x+1} - \sqrt{x-1} = 2$

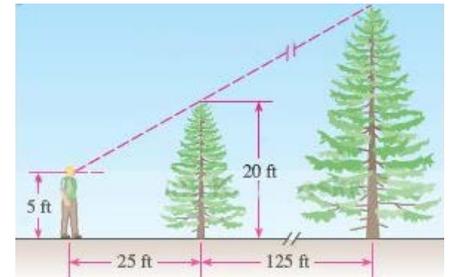
Applications:

Example 6: An experienced painter and an apprentice can paint a room in 6 hours. Working alone, it takes the apprentice 5 hours less than twice the time needed by the experienced painter to paint the room. How long does it take the experienced painter to paint the room?

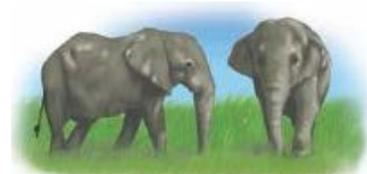
Example 7: A power station is on one side of a river that is 1 mile wide, and a factory is 6 miles downstream on the other side of the river. The cost is \$0.1 million per mile to run power lines over land and \$0.2 million per mile to run power lines under water. How far over land should the power line be run if the total cost of the project is to be \$1 million?



Example 8: A woodcutter determines the height of a tall tree by first measuring a smaller one 125 ft away, then moving so that his eyes are in the line of sight along the tops of the trees, and measuring how far he is standing from the small tree. Suppose the small tree is 20 ft tall, the man is 25 ft from the small tree, and his eye level is 5 ft above the ground. How tall is the taller tree?



Example 9: The average daily food consumption F of a herbivorous mammal with body weight x , where both F and x are measured in pounds, is given approximately by the equation $F = 0.3x^{\frac{3}{4}}$. Find the weight x of an elephant that consumes 300 lb of food per day.



Example 10: A storage bin for corn consists of a cylindrical section made of wire mesh, surmounted by a conical tin roof. The height of the roof is one-third the height of the entire structure. If the total volume of the structure is 1400π ft³ and its radius is 10 ft, what is its height?

