1.5   <u>Inequalities</u>
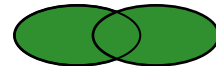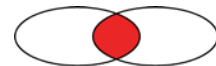
Recall: ∪ **- union** of sets; $A \cup B = \{x | x \in A \ or \ x \in B\}$;

              ∩ **- intersection** of sets; $A \cap B = \{x | x \in A \ \textbf{and} \ x \in B\}$

➢ **linear inequalities**   (solve the same way as equations, except when **multiplying or dividing by a negative** number, **reverse** the inequality sign)

Example 1: Solve, graph, and state your answer in interval notation.
$\frac{1}{3}(2x + 5) - 6 \geq 5x - 8$

➢ **compound inequalities** (watch the "or" or "and" connection)

Example 2: Solve, graph, and state your answer in interval notation.

a) $2x - 5 \leq 3 \ or \ 3x - 4 > 11$ b) $-1 < \frac{2x+5}{4} \leq 3$

c) $2x - 5 \leq 3 \ or \ 3x + 4 > 10$ d) $x - 2 \leq 3 \ and \ 4 - x > 1$

➢ **absolute value inequalities**

$|Expr.| < k$ $|Expr.| > k$

      ⇕

$-k < Expr. < k$ $Expr. < -k \ \ or \ \ Expr. > k$

Example 3: Solve, graph, and state your answer in interval notation.

a) $|1 - 2x| \leq 5$ b) $|4x - 3| > 1$

➢ **polynomial inequalities**
- o make one side $= 0$;
- o factor completely the other side;
- o find all **critical values** (real zeros of the polynomial) to divide the number line into critical intervals;
- o analyse the signs of all factors in all critical intervals;
- o read the answer from the sign diagram

Example 4: Solve using sign analysis.

$x^3 - 3x^2 \geq 10x$

➢ **rational inequalities**
- o make one side $= 0$ and the other a single fraction;
- o factor completely the numerator and denominator;
- o find all **critical values** of the numerator and denominator;
- o divide the number line into critical intervals;
- o analyse the signs of all factors in all critical intervals;
- o read the answer from the sign diagram excluding the numbers that are not in the domain

Example 5: Solve using sign analysis.

a) $\dfrac{x-8}{x-4} < 3$

b) $\dfrac{(5x-3)^2}{2x+1} \leq 0$

Applications:

Example 6: A shoe manufacturer finds that the monthly revenue R from a particular style of aerobics shoes is given by $R = 312p - 3p^2$, where p is the selling price of one pair of shoes. Find the price interval for which the monthly revenue is greater than or equal to $5925.