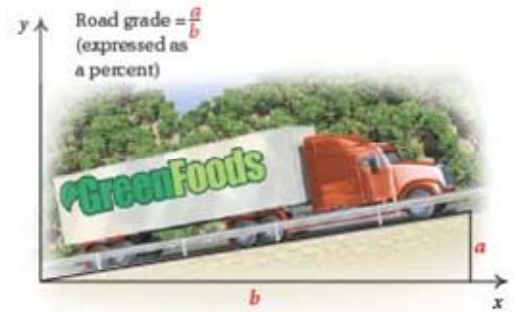
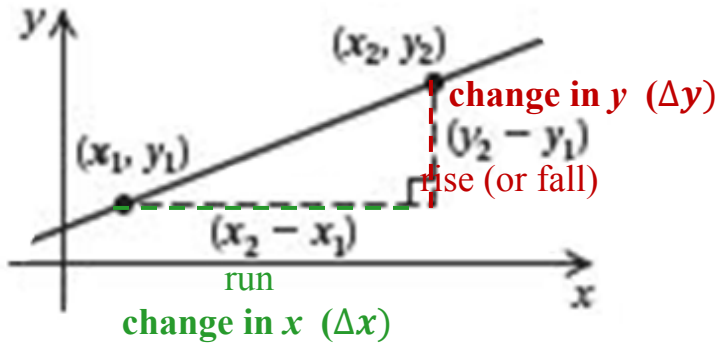


3.2 Slope of a Line and Its Interpretation

slope – steepness of a line; or a **change in elevation in relation to the horizontal distance** between two points; the **rate of change** of y in respect to x ; **rise over run**

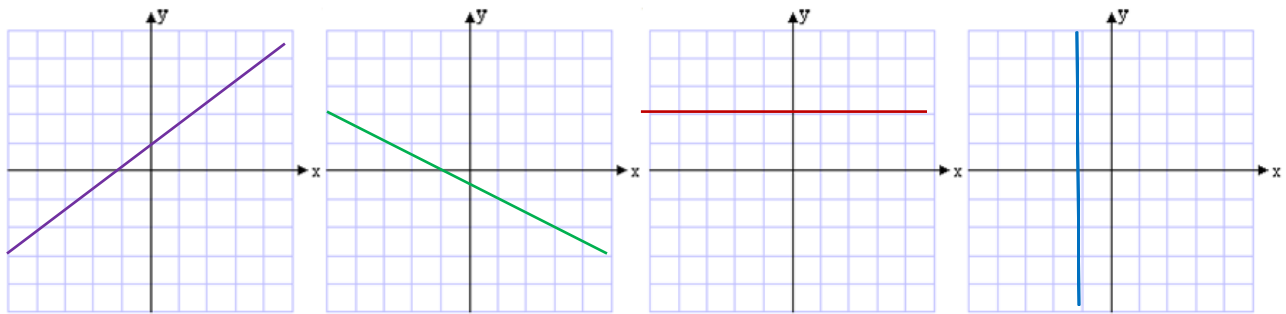


Definition:

Slope m of a line or a line segment connecting two points (x_1, y_1) and (x_2, y_2) is the **difference quotient**:

$$m = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \text{ if } x_1 \neq x_2$$

Example 1: Read the slope of each line given its graph.



Notice: If $x_1 = x_2$, then the points lie on a line and the slope is
 If $y_1 = y_2$ but $x_1 \neq x_2$, then the points lie on a line and the slope is
 If the slope is positive, then the line is
 If the slope is negative, then the line is

Example 2: Calculate the slope of the line passing through each pair of points; then, based on your answer, determine if the line is increasing, decreasing, horizontal, or vertical.

a) $(3,4)$ and $(-2,4)$

b) $(\frac{3}{2}, 4)$ and $(-2, -\frac{1}{4})$

a) $(-2, -5)$ and $(-2, 1)$

b) $(\frac{1}{2}, -\frac{7}{8})$ and $(-\frac{2}{5}, -\frac{1}{4})$

How to find the slope of a line from its equation?

Consider the standard form of a line $Ax + By = C$

What is its **x-intercept**? What is its **y-intercept**?

What is its **slope**?

Now, rewrite the standard form of a line in a slope-intercept form:

What the leading coefficient represents?

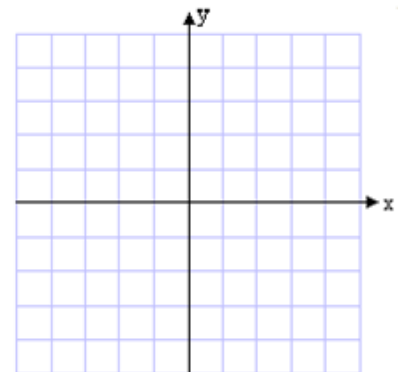
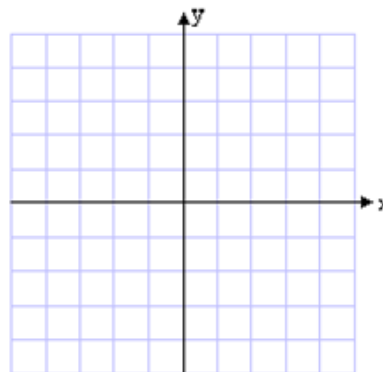
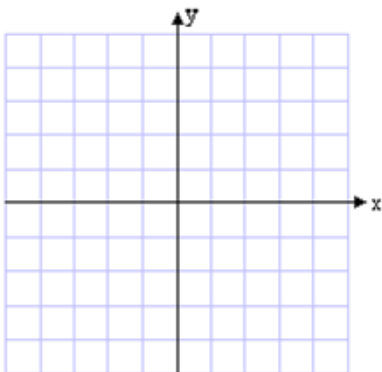
What the constant term represents?.....

Example 3: Find the slope of a line and graph it using slope-intercept method.

a) $2x - 5y = -10$

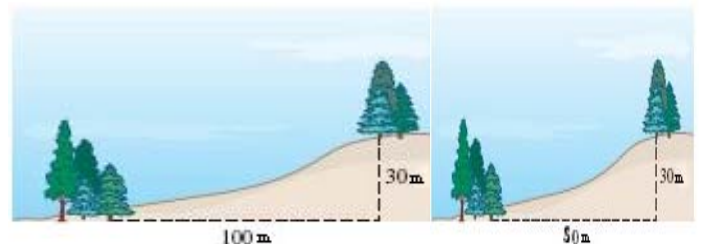
b) $y = 3$

c) $x = -2$

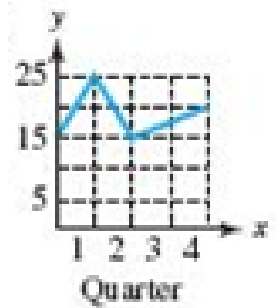


The meaning of slope:

$\text{slope} = \frac{\text{change in } y}{\text{change in } x} = \text{average rate of change}$
in y per unit change in x



Example 4: Consider the graph of sales (in thousands of dollars) during one year.



- a) find the slope of each segment of the graph and describe its meaning
- b) find the average rate of change of sales per year

Parallel and perpendicular lines:

Parallel lines are the lines with the **same steepness**, so they must have the **same slope**.

Practice: 1. Check if the given lines are parallel:

- a) $y = 4x$ and $3y - 12x = 5$
- b) $y = 0$ and $y = 4$
- c) $x = 5$ and $x = -3$

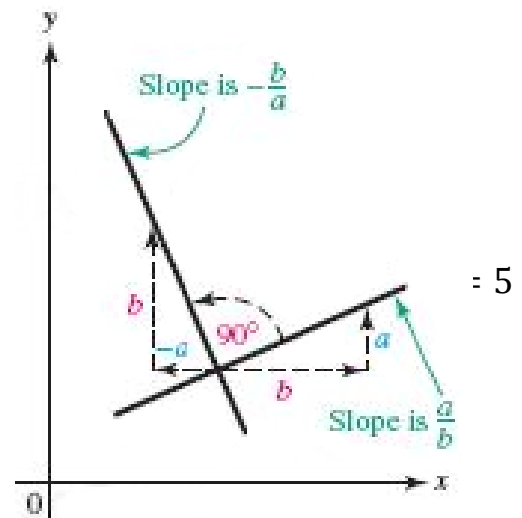
Now, observe how the slopes of perpendicular lines are related:

Conclusion:

Slopes of **perpendicular lines** are **opposite reciprocals**.

Practice: 2. Check if the given lines are perpendicular:

- a) $3y - 6x = 5$ and $2y + x = 5$
- b) $y = x$ and $y = -x$
- c) $x = -1$ and $y = 3$



In summary, if m_1 and m_2 are slopes of two lines, then the lines are:

- parallel iff $m_1 = m_2$, and
- perpendicular iff $m_1 = -\frac{1}{m_2}$ (or equivalently $m_1 \cdot m_2 = -1$)

collinear points – points that lie on the same line; the slope between any pair of such points is the same

Practice:

3. a) Check if the following points are collinear: $A(1, -2)$, $B(3, -1)$, $C(5, 0)$.

b) Does $D\left(\frac{-1}{2}, -\frac{5}{2}\right)$ belongs to the line \overline{AB} ?