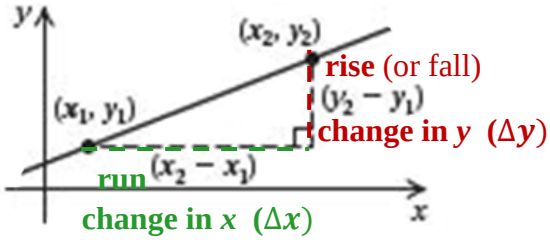
2.3 Linear Function

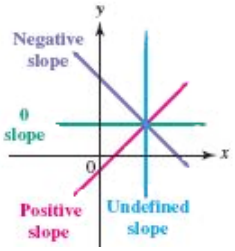
Recall: **Slope *m*** of a line or a line segment connecting two different points (x_1, y_1) and (x_2, y_2) is the **difference quotient**:

$$m = \frac{rise}{run} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}, \quad \text{if } x_1 \neq x_2$$

or ***m*** is **undefined**, if $x_1 = x_2$



slope – **steepness** of a line; or a **change in elevation in respect to the horizontal change**; the **rate of change** of *y* in respect to *x*; **rise over run**



line – a set of points of a plane s.t. the slope between any pair of those points is the same

linear function – any function that can be written in the form $f(x) = mx + b$

Example 1: Find the slope of the line passing through each pair of points.

a) $(3,4)$ and $(-2,4)$

b) $(-5,-1)$ and $(-3,4)$

a) $(-2,-5)$ and $(-2,1)$

b) $\big(1, f(1)\big)$ and $\big(1+h, f(1+h)\big)$

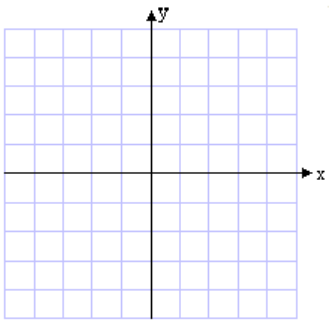
Forms of Linear Equations in Two Variables:

- **slope-intercept** form: $\boldsymbol{y = mx + b}$
- **slope-point** form: $\boldsymbol{y - y_1 = m(x - x_1)}$, where (x_1, y_1) is the given point
- **standard** form: $\boldsymbol{Ax + By = C}$
- **horizontal** line: $\boldsymbol{y = b}$
- **vertical** line: $\boldsymbol{x = a}$

Example 2: Given the equation of the line $3x - 4y = 6$,

A) find its *x*- and *y*-intercepts,

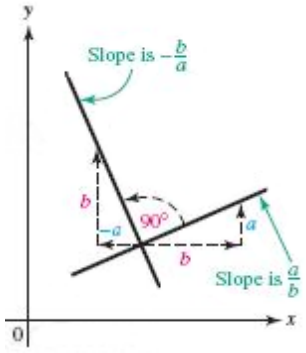
B) rewrite the equation in slope-intercept form and graph it.



Example 3: Write an equation of the line passing through the points $(3,1)$ and $(-1,4)$.

Parallel and perpendicular lines:

Parallel lines are the lines with the **same slope.**
Perpendicular lines have slopes that are **opposite reciprocals**.

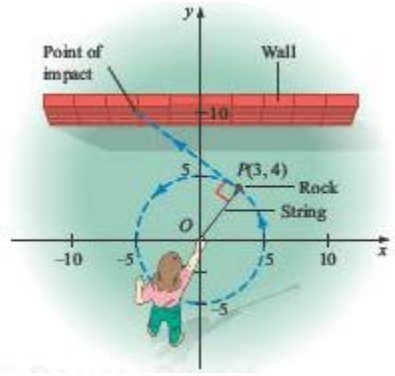


In summary, if $\boldsymbol{m_1}$ and $\boldsymbol{m_2}$ are slopes of two lines, then the lines are:

- parallel iff $\boldsymbol{m_1 = m_2}$, and
- perpendicular iff $\boldsymbol{m_1 = -\frac{1}{m_2}}$ (or equivalently $\boldsymbol{m_1 \cdot m_2 = -1}$)

Example 4: Find the equation of the line whose graph is parallel to the graph of $3x - 2y = 5$ and passes though the point $(-3,4)$.

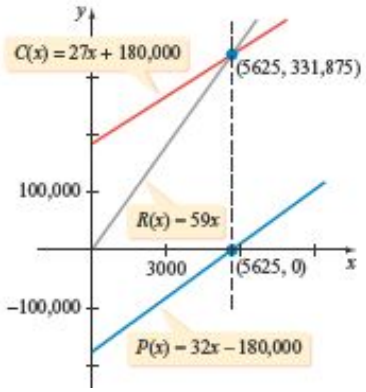
Example 5: A rock attached to a string is whirled horizontally about the origin *O* in a counterclockwise circular path with radius 5 feet. When the string breaks, the rock travels on a linear path perpendicular to the radius *OP* and hits a wall located at y = 10 feet. If the string breaks when the rock is at $P(3,4)$, find the x-coordinate of the point at which the rock hits the wall.



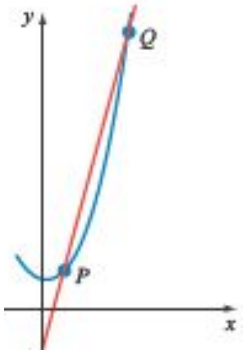
Example 6: A manufacturer finds that the costs incurred in the manufacture and sale of a particular type of calculator are \$180,000 plus \$27 per calculator.

A) Determine the profit function P, given that x calculators are manufactured and sold at \$59 each.

B) Determine the break-even point.



Example 7: Let $P(2,5)$ and $Q(2+h,(2+h)^2+1)$ are two points on the graph of $y = x^2 + 1$.



A) If h = 1, determine the coordinates of Q and the slope of the **secant line** PQ.

B) Do the same for $h = 0.1$

C) Do the same for $h = 0.01$

D) As h approaches 0, what value does the slope of the secant line PQ seem to be approaching?

E) Verify that the slope of the line passing through $(2,5)$ and $(2+h,(2+h)^2+1)$ is $4 + h$.