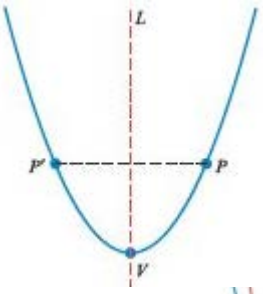
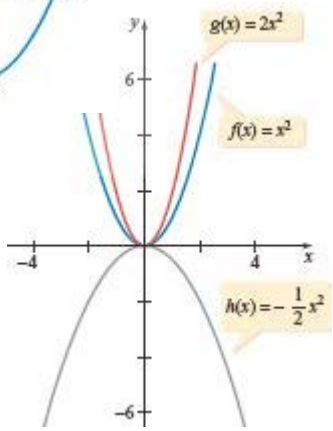
2.4 Quadratic Function

quadratic function – any function that can be written in the form $\boldsymbol{f(x) = ax^2 + bx + c}$, where $a, b, c \in \mathbb{R}$, and $a \neq 0$. This is the **general form** of a **parabola.**



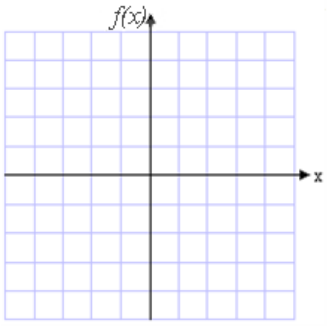
Properties of a Quadratic Function $f(x) = ax^2 + bx + c$



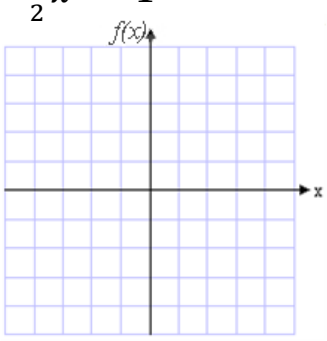
- shape of a **parabola**
- domain = $\mathbb{R}$
- if $a > 0$, the graph opens ……
- if $a < 0$, the graph opens ……
- a is responsible for how narrow or wide the shape is
- **symmetric** with respect to a vertical line passing through the vertex
- can be written in **standard form** $\boldsymbol{f(x) = a(x - h)^2 + k}$, where $h = -\frac{b}{2a}$, and $k = f\left(-\frac{b}{2a}\right) = \frac{4ac-b^2}{4a} = \frac{-\Delta}{4a}$, where $\Delta = b^2 - 4ac$ is the **discriminant**
- coordinates of the **vertex** are $(h, k) = \left(-\frac{\boldsymbol{b}}{\boldsymbol{2a}}, \frac{\boldsymbol{-\Delta}}{\boldsymbol{4a}}\right)$
- the equation of **axis of symmetry** is $x = h$
- if $a > 0$, $\boldsymbol{k}$ is the **minimum** value of *f*, therefore the **range** is $[\boldsymbol{k}, \infty)$
  f **decreases** on $(-\infty, \boldsymbol{h}]$ and **increases** on $[\boldsymbol{h}, \infty)$
- if $a < 0$, $\boldsymbol{k}$ is the **maximum** value of *f*, therefore the **range** is $(-\infty, \boldsymbol{k}]$
  f **increases** on $(-\infty, \boldsymbol{h}]$ and **decreases** on $[\boldsymbol{h}, \infty)$

Example 1: Graph using properties of quadratic function.

A) $f(x) = -2(x - 1)^2 + 3$



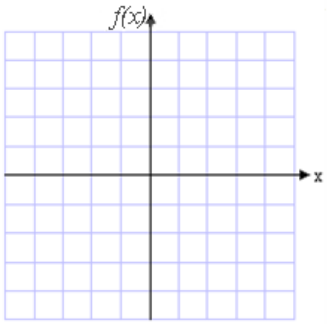
B) $f(x) = \frac{1}{2}x^2 - 1$

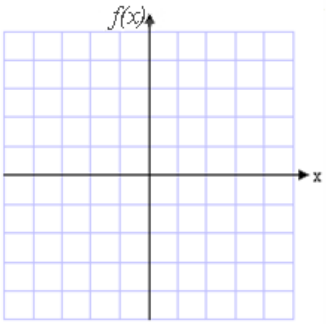


Example 2: Use completing the square procedure to find the standard form of the quadratic function and then graph it.

A) $f(x) = x^2 + 4x + 1$

B) $f(x) = -x^2 - 3x + 1$

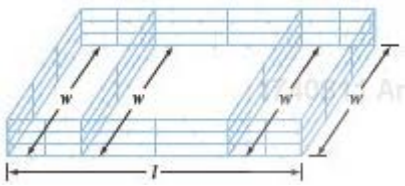




Example 3: Find the range of $f(x) = -x^2 - 8x + 3$ and state the maximum value of *f*.

Example 4: Find the minimum value of $f(x) = 2x^2 + x - 2$ and determine the values of *x* for which $f(x) = 2$.

Example 5: A farmer uses 1200 feet of fence to enclose a rectangular region and to subdivide the region into three smaller rectangular regions by placing fences parallel to one of the sides. Find the dimensions that produce the greatest area.



Example 6: A charter bus company has determined that the cost, in dollars, of providing *x* people with a tour is $C(x) = 180 + 2.50x$. A full tour consists of 60 people. The ticket price per person is \$15 plus \$0.25 for each unsold ticket.

Determine

A) the revenue function

B) the profit function

C) the company's maximum profit

D) the number of ticket sales that yields the maximum profit

Example 7: A Norman window has the shape of a rectangle surmounted by a semicircle. The exterior perimeter of the window is 48 feet. Find the height *h* and the radius *r* that will allow the maximum amount of light to enter the window. (*Hint:* Express the area as a function of *r*.)