4.1 Systems of Linear Equations in Two Variables

solution of a system of equations in x- and y-variables – any ordered pair (x, y) satisfying all the equations of the system;
solution set – the set of all possible solutions (x, y)

Example 1: Check if the point $A(-4,2)$, or $B(3,-12)$ is a solution of the system
$$\begin{cases} 2x + y = -6 \\ x + 3y = 2 \end{cases}$$

To solve a system of equations means to find all possible solutions; that is

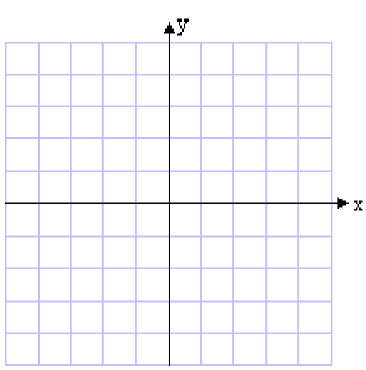
- **all ordered pairs** $(\boldsymbol{x}, \boldsymbol{y})$ satisfying all the equations in the system, or
- **all points** in a plane **that belong to all graphs** of the equations.

How many solutions of the system of two linear equations should we expect ?

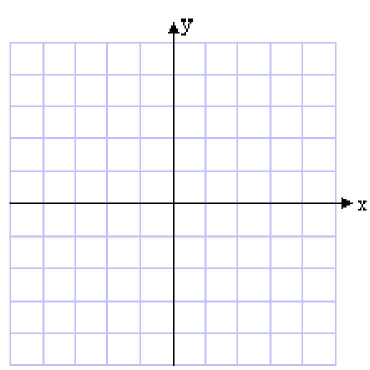
Classification of systems of linear equations:
consistent – if **there is a solution**; otherwise - **inconsistent**
dependent – if the system represents **the same line**; otherwise – **independent**

Example 2: Solve following systems of equations by graphing, and state their solution sets.

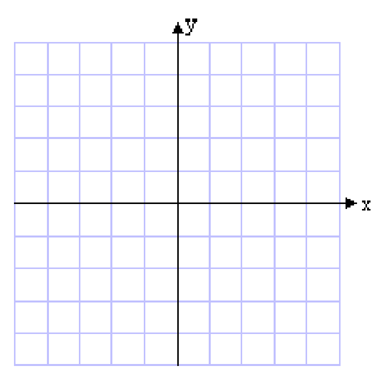
a) $\begin{cases} 2x + y = -5 \\ -x + 3y = 6 \end{cases}$ b) $\begin{cases} 3x - y = -1 \\ 6x - 2y = 2 \end{cases}$ c) $\begin{cases} 2x + y = 5 \\ -6x - 3y = -15 \end{cases}$



solution set: ....



solution set: ....



solution set: ....

So... classify the above systems:
a)

b)

c)

Algebraic methods of solving systems of equations:

- **Substitution Method**

Example 3: Solve $\begin{cases} 5x + y = 7 \\ 3x - 2y = 25 \end{cases}$ by substitution.

Step 1: Solve one of the equations for one of the variables, whichever is easier. This will be your substitution equation.

Step 2: Substitute it to the **<u>other</u>** equation.

Step 3: Solve the resulting equation in one variable.

Step 4: Plug-in the value from step 3 to the substitution equation, to find the value of the other variable.

Step 5: State the answer in the ordered pair form (x, y).

Example 4: Solve $\begin{cases} \frac{1}{2}x + \frac{1}{4}y = \frac{1}{2} \\ \frac{1}{10}x - \frac{3}{5}y = \frac{2}{5} \end{cases}$ by substitution.

Step 1: Clear the fractions by multiplying each equation by LCD if needed.

Step 2: Follow the steps from *example 3*.

➢ Elimination Method

Example 5: Solve $\begin{cases} 8x - 2y = 5 \\ 5x + 2y = -18 \end{cases}$ by elimination.

Step 1: Add equations to eliminate one of the variables (in this case *y*).

Step 2: Solve the resulting equation.

Step 3: Plug-in the value from step 2 to one of the two-variable equations (usually the original equations but not necessarily) to find the other variable.

Step 4: State the answer in the ordered pair form (x, y).

Example 6: Solve $\begin{cases} 2x + 3y = 1 \\ 3x - 5y = -4 \end{cases}$ by elimination.

Step 1: In both equations, **make the coefficients opposite**, by the variable of your choice. This can be done by multiplying one or both equations by appropriate numbers.

Step 2: Follow the steps from *example 5*.

Special cases:

Example 7: Solve $\begin{cases} 4x - 3y = 8 \\ 8x - 6y = 14 \end{cases}$

Solution set = $\emptyset$. The lines are **parallel**.

Example 8: Solve $\begin{cases} 2x + y = 6 \\ -8x - 4y = -24 \end{cases}$

Solution set = $\{(x, y) | 2x + y = 6\}$. The lines are the **same**.

In summary:

If both variables are eliminated in the process of solving a system of equations, then

- we have **no solution**, if the resulting equation is **never true**;
- we have **infinitely many solutions**, if the resulting equation is **always true**.

Example 9: Refer to the graph to answer the following questions:

a) For how many years would the monthly payment be more for the fixed-rate mortgage than for the variable-rate mortgage?

b) When would the payments be the same and what would those payments be? Give an ordered pair of the form (year, payment) to represent this situation.