## 3.1 <u>Division of Polynomials; Remainder and Factor Theorems</u>

Let $P(x)$ be a polynomial of degree $p$ and $D(x)$ a polynomial of degree $d$. If $d \leq p$ then using the **long division algorithm** we can write $\boldsymbol{P(x) = D(x) \cdot Q(x) + R(x)}$, where $Q(x)$ is the **quotient** of degree $q$ and $R(x)$ is the **remainder** polynomial of degree $r$.

What can we say about the degree of the remained $r$?

What can we say about the degree of the quotient $q$?

Recall the **long division algorithm**:

$$x^2 - 2x + 1 \overline{\big) x^4 + x^3 - 10x^2 - 6x - 4}$$

When dividing by a polynomial of the form $\boldsymbol{x - c}$, we can use a shorter form of division, called **synthetic division:**

*Example 1:* Divide $(3x^3 - 2x^2 - 150) \div (x - 4)$

$$x - 4 \overline{\big) 3x^3 - 2x^2 + 0x - 150}$$

$$\begin{array}{r|rrrr} 4 & 3 & -2 & 0 & -150 \\ & \downarrow & 12 & 40 & 160 \\ \hline & 3 & 10 & 40 & 10 \end{array}$$

$$\underbrace{3x^2 + 10x + 40}_{\text{quotient}} + \frac{10}{x-4} \leftarrow \text{remainder}$$

*Example 2:* Divide using synthetic division: $(x^4 - x^3 + 2x + 3) \div (x + 3)$

Generally, when dividing $\boldsymbol{P(x)}$ by $(\boldsymbol{x - c})$, we have $\boldsymbol{P(x) = (x - c) \cdot Q(x) + r}$ , where $\boldsymbol{r}$ is constant. Therefore, $\boldsymbol{P(c)} =$ .........

This proves **The Remainder Theorem**:

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| The remainder in division of $P(x)$ by $(x - c)$ equals to $P(c)$. |

*Conclusion:* **Factor Theorem:**

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| $\boldsymbol{c}$ is a **zero** (root) of a polynomial $P(x)$ iff $P(x)$ is **divisible by** $(\boldsymbol{x - c})$. |

*Example 2:* Given $f(x) = x^4 + 5x^3 + x^2 + 3x - 3$, find $f(-3)$ using synthetic division and the remainder theorem.

*Example 3:* Factor $f(x) = 6x^3 + 19x^2 + 2x - 3$ into linear factors if $-3$ is a zero of $f$.

*Example 4:* Determine the value of $k$ that will make $P(x) = 2x^3 + x^2 + kx + 5$ divisible by $x + 1$.