

5.1 Integer Exponents and Scientific Notation

Recall:

$$a^n = \underbrace{a \cdot a \cdot \dots \cdot a}_{n \text{ times}}$$

exponential rules:

$$a^m \cdot a^n = a^{m+n}$$

$$\begin{cases} a^0 = 1, & \text{if } a \neq 0 \\ 0^0 \text{ is undefined} \end{cases}$$

$$\frac{a^m}{a^n} = a^{m-n}$$

$$a^{-1} = \frac{1}{a}, \quad \text{if } a \neq 0$$

$$(a^m)^n = a^{m \cdot n}$$

$$a^{-n} = \left(\frac{1}{a}\right)^n = \frac{1}{a^n}, \quad \text{if } a \neq 0$$

$$(ab)^n = a^n b^n$$

$$\text{so } \left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^n, \quad \text{if } a, b \neq 0$$

$$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$$

$$\text{and } \frac{a^{-n}}{b^{-m}} = \frac{b^m}{a^n}, \quad \text{if } a, b \neq 0$$

Practice: 1. Simplify. Leave the answer with **positive exponents**. Avoid evaluating large numbers. Leave them as **powers of primes**.

a) $3x^2(-5x^{-5}) =$

b) $1 - (-2)^0 =$

c) $-5^{-2} =$

d) $-5x^{-2} =$

e) $(-5x)^{-2} =$

f) $\left(\frac{2^{-1}+3^{-1}}{4^{-1}}\right)^{-1} =$

g) $\frac{5^{-6}}{5^{-15}} =$

h) $\left(\frac{-2a}{b^3}\right)^2 \left(\frac{4b}{a}\right)^{-3} =$

i) $\left(\frac{-3x^4y^{-2}}{15x^{-6}y^5}\right)^{-3} =$

$$j) \left(\frac{-200x^3y^{-5}}{8x^5y^{-7}} \right)^{-4} =$$

$$k) \left(\frac{a^6b^{-2}}{2a^{-2}} \right)^{-1} \cdot \left(\frac{6a^{-2}}{5b^{-4}} \right)^2 \cdot \left(\frac{2b^{-1}a^2}{3b^{-2}} \right)^{-1} =$$

$$l) x^{2a+1}x^{a-1} =$$

$$m) \frac{y^{2m}}{y^{m-1}} =$$

$$n) (t^{a+1})^{a-1} =$$

Scientific Notation:

Definition: A number x is written in **scientific notation** iff $x = a \cdot 10^n$,
 where $|a| \in [1,10)$, and $n \in \mathbb{Z}$.
 \uparrow
coefficient

Example 1: Write each number in scientific notation.

$$a) 967000000 =$$

$$b) 0.000459 =$$

$$c) 12.045 \cdot 10^{-3} =$$

$$d) 0.06 \cdot 10^6 =$$

Notice: The sign of the exponent tells us which way to move the decimal point of the **coefficient a** to find the actual value of the number x .

Example 2: Write each number in decimal notation.

$$a) 3.05 \cdot 10^{-5} =$$

$$b) 5.107 \cdot 10^4 =$$

Example 3: Perform each operation and leave the answer in scientific notation.

$$a) 8.5 \cdot 10^{11} \cdot 2.0 \cdot 10^7 =$$

$$b) \frac{1.5 \cdot 10^{24}}{6.0 \cdot 10^8} =$$

Example 4: The average ocean depth is $3.7 \cdot 10^3$ m, and the area of the oceans is $3.6 \cdot 10^{14}$ m². What is the total volume of the oceans in litres? ($1\text{m}^3 = 1000\text{L}$)

