

5.2 Addition and Subtraction of Polynomials

term – a product of numbers, letters, and possibly algebraic expressions

ex. $-2xy^2$, $\frac{2}{5}x(x-1)$, πx , $5a^{-2}(a^2 - b^2)$, $2\sqrt{x}$

monomial – a term that is a product of a number (called **numerical coefficient**) and some letters raised to nonnegative integers; a term in one variable can be written in the form ax^n , where a is the **numerical coefficient**, x is the variable, and n is the nonnegative exponent.

ex. $-2xy^2$, πx , -1 , $\frac{1}{2}xyz$, $5x^{16}$

binomial – sum of two monomials; ex. $2x^2 - 3x$, $x + 1$

trinomial – sum of three monomials; ex. $x^2 - x + 1$, $-xyz + 2x^2 - y$

polynomial – sum of monomials; any polynomial in one variable can be written in the form:

$$a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$$

degree of a term (monomial) – the sum of the exponents of all the variables

ex. $\deg(-2xy^2) = \dots\dots\dots$ $\deg\left(\frac{1}{2}xy^3z^2\right) = \dots\dots\dots$ $\deg(-2) = \dots\dots\dots$

degree of a polynomial – the greatest degree of all of its terms

ex. $\deg(-xyz + 2x^2 - y) = \dots\dots\dots$ $\deg(5xy + 2x^3y - y^3) = \dots\dots\dots$

leading term – the greatest-degree term; $a_n x^n$

leading coefficient – the coefficient of the leading term; a_n

Polynomials are traditionally written in **descending powers** of the variable. For polynomials with more than one variable, the degrees of terms and alphabetical order of variables determine the order of terms.

ex. $1 - x^4 + 2x^2 - x^3 + 3x$ should be written as $\dots\dots\dots$
 $2p^2q^3 - pq^2 - 3p^3 + q^2$ should be written as $\dots\dots\dots$

like terms – terms with the same variables to the same exponents

ex. $-2xy^2$, $\sqrt{2}xy^2$, $5xy^2$ but not $2x^2y$, $-x^3$, $3xy$

Like terms can be added (**combined**) by adding their coefficients: $2x + 3x =$

Subtraction is defined as addition of opposite: $a - b = a + (-b)$

So, think of polynomials as sums of signed terms: $2x - 3x =$

Example 1: Perform the operations and simplify.

a) $-(a - b) =$

b) $-(-2x + 3y - 1) =$

c) $(2x^2 - 3x + 6) + (-4x^2 + x - 3) =$

d) $(2x^2 - 3x + 6) - (-4x^2 + x - 3) =$

e) $-[5y^2 + 2y - (3y^2 - 4y)] + [(7y^3 - [y - 2y^3]) + 3y^2] =$

f) subtract $x^{2a} - 3x^a + 1$ from $x^{3a} + 3x^a - 2$