

3.3, 3.4 In-class Practice

1. If the polynomial $P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_1 x + a_0$ has integer coefficients, then the only numbers that could possibly be rational zeros of P are all of the form $\frac{p}{q}$, where p is a factor of _____ and q is a factor of _____.

The possible rational zeros of $P(x) = 6x^3 + 5x^2 - 19x - 10$ are

2. Find all rational zeros of the polynomial, then write the polynomial in factored form.

a) $P(x) = x^3 + 3x^2 - 4$

b) $P(x) = 3x^3 + 17x^2 + 21x - 9$

c) $P(x) = x^4 - 5x^3 + 6x^2 + 4x - 8$

d) $P(x) = 2x^4 + 15x^3 + 17x^2 + 3x - 1$

3. Show that the polynomial $P(x) = x^{50} - 5x^{25} + x^2 - 1$ does not have any rational zeros.

4. **CONCEPTS:**
- A.** The polynomial $P(x) = 3(x - 5)^3(x - 3)(x + 2)$ has degree _____. It has zeros 5, 3, and _____. The zero 5 has multiplicity _____, and the zero 3 has multiplicity _____.
- B.** (a) If a is a zero of the polynomial P , then _____ must be a factor of $P(x)$.
(b) If a is a zero of multiplicity m of the polynomial P , then _____ must be a factor of $P(x)$ when we factor P completely.
- C.** A polynomial of degree $n \geq 1$ has exactly _____ zeros if a zero of multiplicity m is counted m times.
- D.** If the polynomial function P has real coefficients and if $a + bi$ is a zero of P , then _____ is also a zero of P .

5. Find all zeros (real and complex) of the polynomial P and then factor it completely.

a) $P(x) = x^4 + 6x^2 + 9$

b) $P(x) = x^3 - 8$

6. Factor the polynomial completely and state its zeros, including multiplicity.

a) $P(x) = x^4 + 3x^2 - 4$

b) $P(x) = x^5 + 7x^3$

7. Find a polynomial with integer coefficients satisfying the following conditions:

a) P has degree 2 and zeros $1 + i$ and $1 - i$

b) S has degree 4 and zeros $2i$ and $3i$

c) T has degree 4, zeros i and $1 + i$, and constant term 12

d) Q has degree 5, zeros $\frac{1}{2}$, -1 (of multiplicity 2), and $-i$, and leading coefficient 4

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8. Find all solutions of the equation.

a) $x^2 - ix = 0$

b) $x^2 + 2ix - 1 = 0$

9. One zero is given. Find all other zeros.

a) $f(x) = x^4 + 5x^2 + 4$; $-i$

b) $f(x) = x^3 - 7x^2 + 17x - 15$; $2 - i$