3.3, 3.4 Zeros of Polynomial Functions and Fundamental Theorem of Algebra

How to find rational zeros of a polynomial with integer coefficients?

Rational Zero Theorem
Let $\boldsymbol{P(x) = a_n x^n + \cdots + a_1 x + a_0}$ be a polynomial with integer coefficients $(a_n \neq 0)$.
If a reduced rational number $\boldsymbol{\frac{p}{q}}$ is a root of the polynomial *P*, then

– $\boldsymbol{p}$ divides the constant coefficient $\boldsymbol{a_0}$, and
– $\boldsymbol{q}$ divides the leading coefficient $\boldsymbol{a_n}$**.**

Example 1: use Rational Zero Theorem to find all possible rational zeros of

a) $P(x) = x^3 - 2x^2 - 5x + 6$

b) $P(x) = 6x^4 + 23x^3 + 19x^2 - 8x - 4$

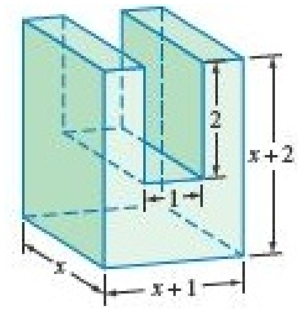
Example 2: Find the zeros of each polynomial. State the multiplicity of each zero that is not simple. (*Def.:* **simple zero -** zero of multiplicity one)

a) $P(x) = x^3 - 4x^2 - 3x$

b) $P(x) = x^3 + 3x^2 - 6x - 8$

c) $P(x) = 2x^4 - 17x^3 + 4x^2 + 35x - 24$

Example 3: For what value of x will the volume of the given solid be 112 cubic inches?



Fundamental Theorem of Algebra

Any n^{th} degree $(n \geq 1)$ polynomial P with complex coefficients has at least one complex zero.

This means that any polynomial $P(x) = a_n x^n + \cdots + a_1 x + a_0$, where $n \geq 1$, $a_n \neq 0$, and $a_n, a_{n-1}, \ldots, a_0 \in \mathbb{C}$, can be expressed in a factored form

$$\boxed{P(x) = a_n(x - \boldsymbol{c_1})(x - \boldsymbol{c_2}) \cdot \ldots \cdot (x - \boldsymbol{c_n})},$$

where $c_1, c_2, \ldots, c_n$ are some complex roots of P, not necessarily different.

Example 4: Solve the equation (*in complex numbers, if needed*).

a) $5x^3 - 6x^2 - 29x + 6 = 0$

b) $x^3 - 3x^2 + 4x - 12 = 0$

Conjugate Rule for Polynomials with Real Coefficients

If a complex number $\boldsymbol{a+bi}$ (where $b \neq 0$) is a zero of a polynomial with real coefficients, then the conjugate $\boldsymbol{a-bi}$ is also a zero of this polynomial.

Example 5: Find a polynomial P with real coefficients, satisfying the following conditions:

a) P is a 3rd degree polynomial; $-2, 1-i$ are zeros of P

b) P is a 4th degree polynomial; i is a zero of P with multiplicity 2; $P(1) = 2$