5.3 Polynomial Functions, Graphs, and Composition

Any **polynomial function of degree *n*** can be written in the form

$$\boxed{\boldsymbol{P(x) = a_n x^n + a_{n-1} x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0}},$$

where $a_n, a_{n-1}, \ldots a_2, a_1, a_0 \in \mathbb{R}$, $n \in \mathbb{W}$, and $a_n \neq 0$.

Polynomials can be evaluated for every $x \in \mathbb{R}$,
so the domain of any polynomial is ....

Example 1: Given $f(x) = -2x^2 - x + 5$, evaluate

a) $f(-1)$ b) $f(2)$

c) $f\left(\frac{1}{2}\right)$ d) $2\,f(x)$

e) $f(x) - 5$ f) $f(a)$

g) $f(a+1)$

The **set of polynomials** is **closed under** operations such as **addition, subtraction, multiplication**, and **composition**. That means that the result of such operations is also a polynomial.

Definition 1: For any given function $\boldsymbol{f}$ (with the domain D_f), and $\boldsymbol{g}$ (with the domain D_g), we can define the **sum**: $\boldsymbol{(f+g)(x) = f(x) + g(x)}$, and
the **difference**: $\boldsymbol{(f-g)(x) = f(x) - g(x)}$,

The **domain** of the sum or difference function is the intersection $\boldsymbol{D_f \cap D_g}$ of the domains of functions $\boldsymbol{f}$ and $\boldsymbol{g}$.

Example 2: Let $P(x) = 2x - 1$, and $S(x) = x^2 + 2x - 1$. Find the following:

a) $(P+S)(3)$

b) $(P+S)(x)$

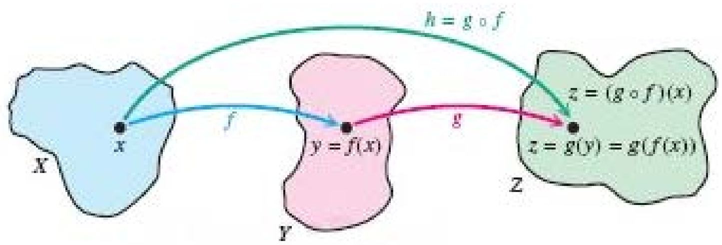
c) $(S-P)(x)$

d) $(S-P)(-1)$

e) $(P-S)(a)$

Definition 2: For any given function $\boldsymbol{f}$ and $\boldsymbol{g}$, we can define the **composition**:

$(\boldsymbol{g} \circ \boldsymbol{f})(\boldsymbol{x}) = \boldsymbol{g}(\boldsymbol{f}(\boldsymbol{x}))$ as long as $f(x)$ is in the domain of $\boldsymbol{g}$.



For polynomial functions, the domain of a composition is also $\mathbb{R}$.

Example 3: Given $f(x) = 3x - 2$, and $g(x) = x^2 + 1$, find the following:

a) $(g \circ f)(2)$

b) $(f \circ g)(2)$

c) $(g \circ f)(x)$

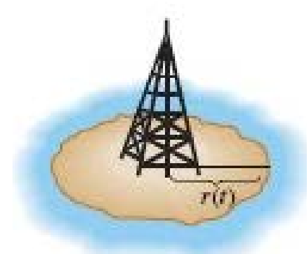
d) $(f \circ g)(x)$

e) $(f \circ f)(0)$

f) $(f \circ f)(x)$

g) $(g \circ g)(-1)$

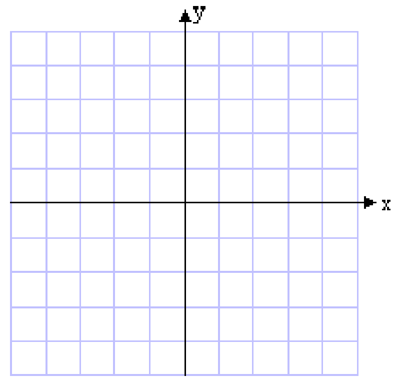
Example 4: An oil well is leaking, with the leak spreading oil over the surface as a circle. At any time $\boldsymbol{t}$, in minutes, after the beginning of the leak, the radius of the circular oil slick on the surface is $r(t) = 4t$ feet. Let $A(r) = \pi r^2$ represent the area of a circle of radius r. Find and interpret $(A \circ r)(t)$.



Graphs of basic polynomials:

Example 5: Using a table of values, **graph** each function and then give its **domain** and **range**.

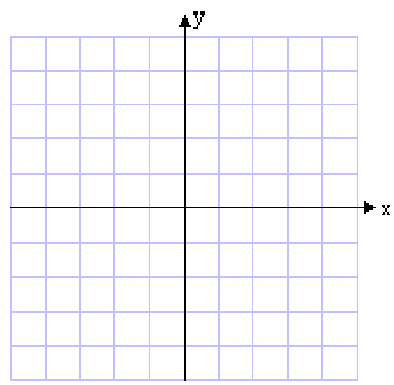
a) $f(x) = 2$
(**constant** function)



x	$f(x) = 2$

Domain: ....
Range: ....

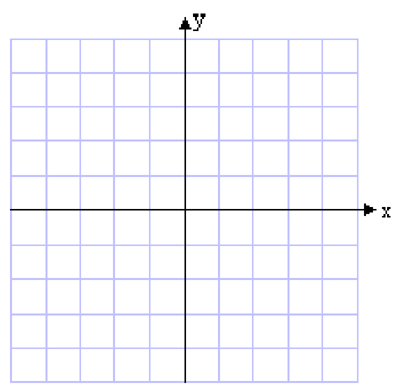
b) $f(x) = x$
(**identity** function)



x	$f(x) = x$

Domain: ....
Range: ....

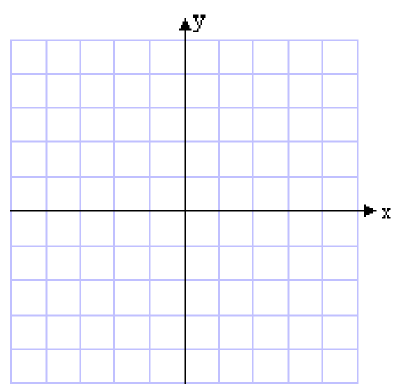
c) $f(x) = 2x - 1$
(**linear** function)



x	$f(x) = 2x - 1$

Domain: ....
Range: ....

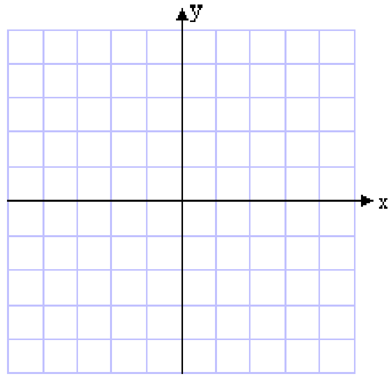
d) $f(x) = x^2$
(**basic quadratic** function)
parabola



x	$f(x) = x^2$

Domain: ....
Range: ....

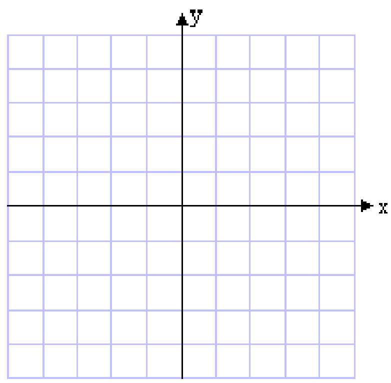
e) $f(x) = -x^2$
(**quadratic** function)



x	$f(x) = -x^2$

Domain: ....
Range: ....
Transformation of basic shape: ....

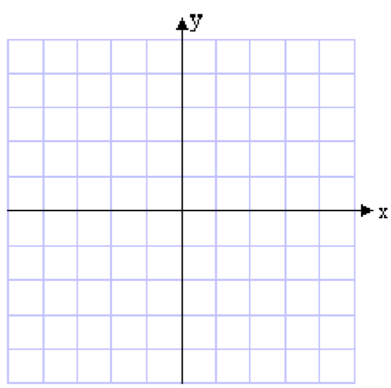
f) $f(x) = x^2 - 4$
(**quadratic** function)



x	$f(x) = x^2 - 4$

Domain: ....
Range: ....
Transformation of basic shape: ....

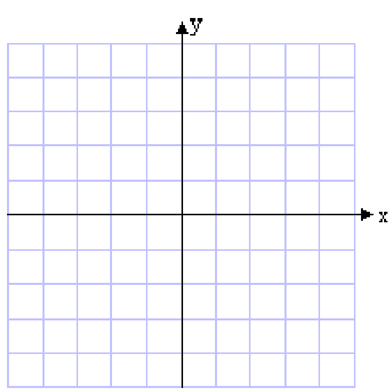
g) $f(x) = (x - 2)^2$
(**quadratic** function)



x	$f(x) = (x - 2)^2$

Domain: ....
Range: ....
Transformation of basic shape: ....

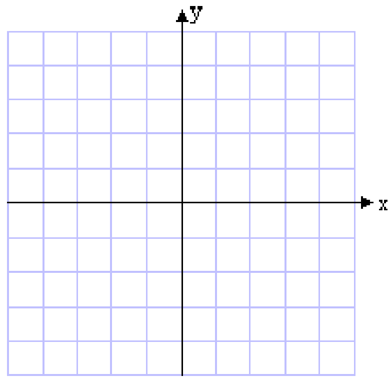
h) $f(x) = x^3$
(**basic cubic** function)



x	$f(x) = x^3$

Domain: ....
Range: ....

i) $f(x) = x^3 + 1$
(**cubic** function)



x	$f(x) = x^3 + 1$

Domain: ....
Range: ....
Transformation of basic shape: ....