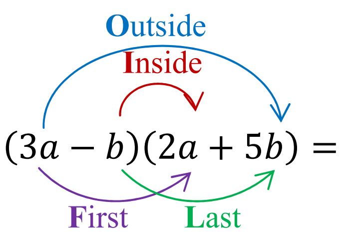
5.4 Multiplication of Polynomials

To multiply monomials, multiply numerical coefficients and add exponents of the same variables:

$2x^2y(-3x^3y^5) =$

To multiply polynomials, multiply each term of one polynomial by each term of the other polynomial and then collect like terms:

$-2x^3y^2(3x^2 - 5xy^2 + 1) =$



$(3a - b)(2a + 5b) =$

$(2x^2 - x + 1)(3x^2 + 4x - 5) =$

$4x^3(2x + 1)(x - 2) =$

Try to multiply binomials more efficiently by adding inner and outer products in your mind:

$(x + 5)(x + 2) =$

$(2x - 3)(x + 5) =$

Special Products:

Perfect Square: $(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} + \boldsymbol{b}) = (\quad)^2 =$

$(\boldsymbol{a} - \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b}) = (\quad)^2 =$

Difference of Squares: $(\boldsymbol{a} + \boldsymbol{b})(\boldsymbol{a} - \boldsymbol{b}) =$

Remember! $(\boldsymbol{x} + \boldsymbol{y})^2 \neq \boldsymbol{x}^2 + \boldsymbol{y}^2$ and generally $(\boldsymbol{x} + \boldsymbol{y})^n \neq \boldsymbol{x}^n + \boldsymbol{y}^n$ for $n \neq 1$.

Example 1: Multiply polynomials, using special product formulas when appropriate.

a) $(2x + 3y)(2x - 3y)$

b) $(2x + 3y)^2$

c) $(.2a - .5b^2)(.2a + .5b^2)$

d) $\left(3x - \frac{1}{2}y\right)^2$

f) $(x^a - y^b)^2$

e) $(x + y - 2)(x + y + 2)$

f) $(3a + b)(3a - b)(9a^2 - b^2)$

g) $(2x + y - 3z)^2$

h) $(x - 2)^3$

i) $(p + 1)^4$

j) $(5 - 2n + n^2)(5 + 2n - n^2)$

Example 2: Find the product $298 \cdot 302$ using difference of squares formula.

As we see, polynomial functions can be multiplied. Generally, we have:

Definition 1: For any given function $\boldsymbol{f}$ (with the domain D_f), and $\boldsymbol{g}$ (with the domain D_g), we can define the **product function**: $(\boldsymbol{f} \cdot \boldsymbol{g})(\boldsymbol{x}) = \boldsymbol{f}(\boldsymbol{x}) \cdot \boldsymbol{g}(\boldsymbol{x})$
The **domain** of the product function is the intersection $\boldsymbol{D_f} \cap \boldsymbol{D_g}$ of the domains of functions $\boldsymbol{f}$ and $\boldsymbol{g}$.

Notice: $(f \cdot g)(x)$ or simply $(fg)(x)$ represents the product function and it is not the same as $(f \circ g)(x)$ or $f\big(g(x)\big)$ which represent the composition of functions.

Example 3: Given $f(x) = 3x - 2$, and $g(x) = x^2 + 1$, find the following:

a) $(fg)(x)$

b) $(fg)(-1)$

c) $2 \cdot (fg)(0)$

Example 4: Find the area of the given triangle.