5.5 Division of Polynomials

To divide a polynomial by a **monomial**, we rewrite the expression into **simple fractions** according to the rule $\boxed{\frac{a \pm b}{d} = \frac{a}{d} \pm \frac{b}{d}}$ and then **simplify** within each fraction.

Example 1: Divide.

a) $\frac{6x^3 - 9xy^2}{18xy}$

b) $\frac{12x^4y^3 - 15xy^2 + 18x^5y^4}{-6x^2y^3}$

To divide a polynomial by a polynomial, use **long division algorithm**.
Example 2: Divide $-4x^2 + 3x + 5$ by $x - 2$.

Remember to arrange both polynomials in decreasing order of exponents and replace all the missing terms by zero!
Example 3: Find the quotient $\frac{-3x^3 + 4x - 1}{x^2 + 3x}$.

Observation: The remainder is always of lower degree than the divisor.

Example 4: Let $P(x) = 3x^3 + 4x^2 + 7x + 4$ and $D(x) = 3x + 3$. Use division to find polynomials $Q(x)$ and $R(x)$ such that $P(x) = Q(x) \cdot D(x) + R(x)$.

Definition 1: For any given function ***f*** (with the domain D_f), and ***g*** (with the domain D_g), we can define the **quotient function**: $\boldsymbol{\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}}$

The **domain** of the quotient function is $\boldsymbol{D_f \cap D_g} \setminus \{\boldsymbol{x} \in \mathbb{R} |\ \boldsymbol{g(x) = 0}\}$, the intersection of the domains of functions ***f*** and ***g***, excluding all ***x***-values such that $g(x) = 0$.

Example 5: Let $f(x) = x^2 - 4$, $g(x) = x - 2$, $h(x) = -2x$. Find the following:

a) $\left(\frac{f}{h}\right)(x)$

b) the domain of $\left(\frac{f}{h}\right)(x)$

c) $\left(\frac{f}{g}\right)(x)$

d) the domain of $\left(\frac{f}{g}\right)(x)$

e) $\left(\frac{f}{h}\right)(-1)$

f) $\left(\frac{f}{g}\right)(-1)$

g) $\left(\frac{f}{h}\right)(0)$

h) $\left(\frac{f}{g}\right)(2)$

i) $\left(\frac{g}{h}\right)\left(\frac{1}{2}\right)$

j) $\left(\frac{h}{g}\right)\left(\frac{3}{2}\right)$