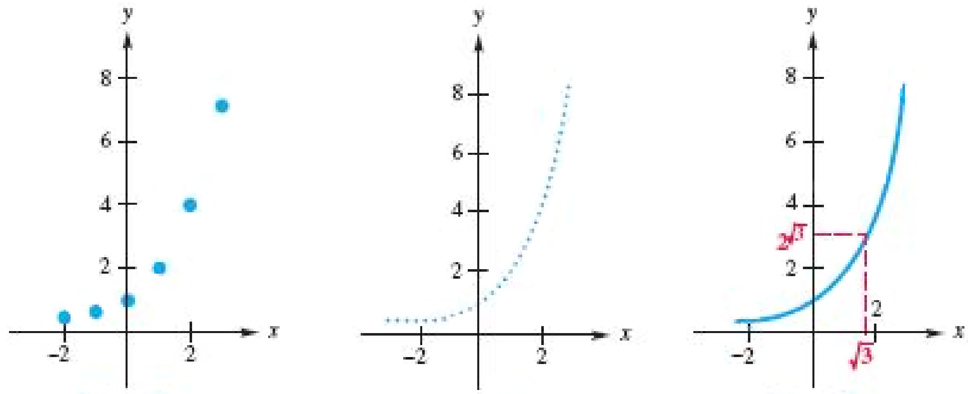
4.2 <u>Exponential Functions and Their Applications</u>

Definition: The **exponential function** with base ***b*** (***b*** > 0 and $b \neq 1$) is defined by $f(x) = b^x$ for all real numbers ***x***.



$f(x) = 2^x$; integers as domain

$f(x) = 2^x$; selected rational numbers as domain

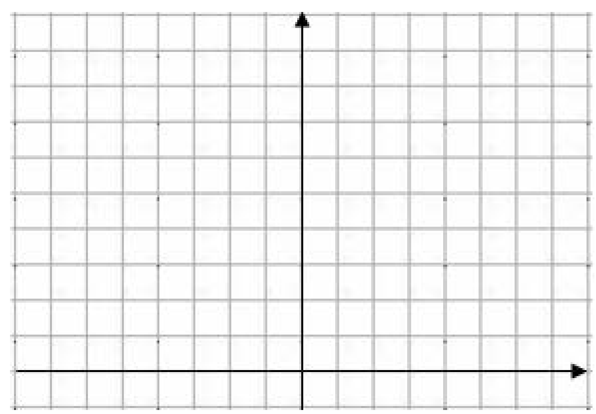
$f(x) = 2^x$; real numbers as domain

Recall some properties of exponents: $a^{-n} = \frac{1}{a^n}$, $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$

For example calculate: $2^{\frac{5}{2}} =$ $\left(\frac{1}{2}\right)^{-2} =$

Example 1: Using table of values, graph the following functions on the same grid and observe their properties:

$f(x) = 2^x$, $g(x) = 3^x$, $h(x) = \left(\frac{1}{2}\right)^x$

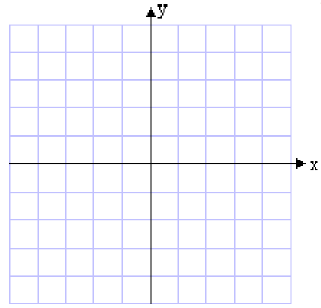


Properties of an exponential function $f(x) = b^x$:

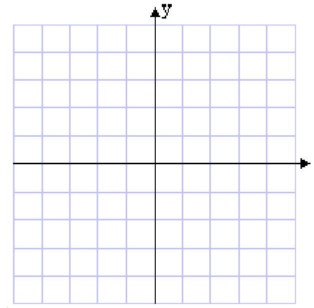
- Domain = Range =
- If $\boldsymbol{b} > 1$, then the function is **increasing**; so if $n < m$ then $b^n < b^m$.
- If $\mathbf{0} < b < 1$, then the function is **decreasing**; so if $n < m$ then $b^n > b^m$.
- It has ***y*-intercept** at $(\mathbf{0}, \mathbf{1})$, no *x*-intercept, and the graph passes through $(\mathbf{1}, \boldsymbol{b})$**.**
- It is a **one-to-one** function; so $b^n = b^m$ iff $n = m$
- It has a **horizontal asymptote** ***y*** = 0.

Example 2: Use transformation of the graph of $f(x) = 2^x$ to produce a graph of

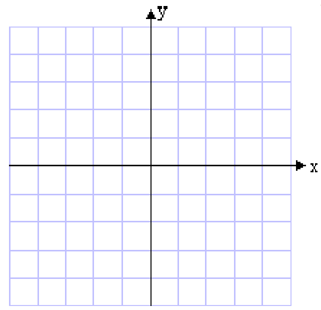
a) $g(x) = -2^{x-3} + 1$



b) $h(x) = \frac{1}{2} \cdot 2^{x+1} - 3$



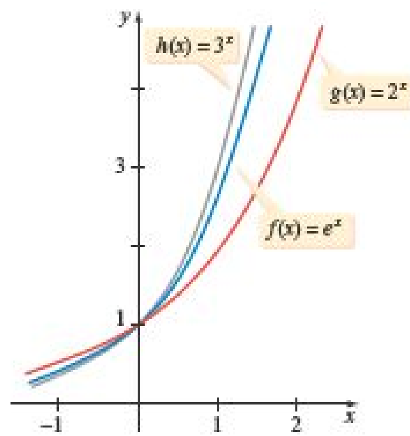
c) $p(x) = -2^{-x} + 2$



Exponential function is used in many applications that involve growth or decay. The most commonly used base is the irrational constant $e \approx 2.71828$.

Definition: The number $\boldsymbol{e}$ is the limiting value of the expression $\left(\mathbf{1} + \frac{\mathbf{1}}{\boldsymbol{n}}\right)^{\boldsymbol{n}}$, when $n \to \infty$.

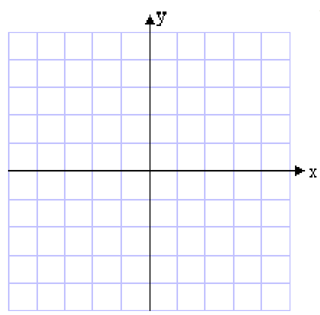
Also, the number e is a base of an exponential function that produces the graph with the slope of the tangent line at $(0,1)$ equal to 1.

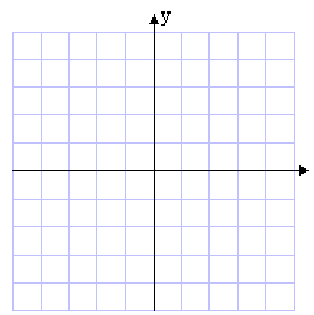


Example 3: Using a graphing calculator, graph the following functions and observe their properties, including their asymptotic behavior.

a) $f(x) = \frac{e^x + e^{-x}}{2}$

b) $g(x) = \frac{5}{1 + e^{-0.5x}}$





Example 4: Determine the domain of

a) $f(x) = \frac{1}{1-e^x}$

b) $f(x) = \sqrt{1 - e^{2x}}$

Example 5: The exponential function $A(t) = 200e^{-0{,}014t}$ gives the amount of medication, in milligrams, in a patient's bloodstream ***t*** minutes after the medication has been injected into the patient's bloodstream.

a) Find the amount of medication, to the nearest milligram, in the patient's bloodstream after 45 minutes.

b) Using a graphing calculator, determine how long it will take, to the nearest minute, for the amount of medication in the patient's bloodstream to reach 50 milligrams.

Example 6: The number of bass in a lake is given by $P(t) = \frac{3600}{1+7e^{-0.05t}}$, where t is the number of months that have passed since the lake was stocked with bass.

a) How many bass were in the lake immediately after it was stocked?

b) How many bass, to the nearest one, were in the lake 1 year after the lake was stocked?

c) After how many months we should expect to have 2000 bass in the lake?

d) What will happen to the bass population as t increases without bound?