6.1-6.4 Factorization of Polynomials

to factor – to rewrite as a product; ex. $x^2 + 4x - 21 = (x - 3)(x + 7)$
prime polynomial – a polynomial that can't be expressed as a product of other polynomials with integer coefficients; ex. $2x + 3$, $x^2 + 1$, $x^2 + x + 1$
to factor a polynomial **completely** – to rewrite the polynomial as a product of prime polynomials;

Factoring Strategies:

1. **Common Factor**

   $-12x^4y + 16x^3y^2 - 8xy^3 =$

   $2x(x - 1) + 3y(x - 1) =$

   $(2x + 1)(x + 5) - (2x + 1)(x - 7) =$

   $-2a^{-3} + 6a^{-5} - 14a^{-4} =$

2. **Factoring by Grouping**

   $6xy - 4y + 15x - 10 =$

   $5xy - x^2y - 10 + 2x =$

3. **Factoring Trinomials with the Leading Coefficient = 1** (guessing method)

$$x^2 + \underbrace{\boldsymbol{b}}_{(\boldsymbol{p}+\boldsymbol{q})} x + \underbrace{\boldsymbol{c}}_{\boldsymbol{pq}} = (x + p)(x + q)$$

Factor the following polynomials and observe the role of signs by the free term and by the middle term:

$x^2 + 5x + 6 =$ $\qquad$ $x^2 - 5x + 6 =$

$x^2 + x - 6 =$ $\qquad$ $x^2 - x - 6 =$

In summary, assuming that $|\boldsymbol{p}| \geq |\boldsymbol{q}|$:

$+\ + \Rightarrow\ \boldsymbol{p}, \boldsymbol{q} > 0$ and $\boldsymbol{b} = \boldsymbol{p} + \boldsymbol{q}$	$-\ + \Rightarrow \boldsymbol{p}, \boldsymbol{q} < 0$ and $\boldsymbol{b} = \boldsymbol{p} + \boldsymbol{q}$
$+\ - \Rightarrow\ \boldsymbol{p} > 0,\ \boldsymbol{q} < 0$ and $\boldsymbol{b} = \lvert\boldsymbol{p}\rvert - \lvert\boldsymbol{q}\rvert$	$-\ - \Rightarrow \boldsymbol{p} < 0,\ \boldsymbol{q} > 0$ and $\boldsymbol{b} = \lvert\boldsymbol{q}\rvert - \lvert\boldsymbol{p}\rvert$

Example 1: Factor completely.

Remember:
- always look for the common factor first;
- arrange the polynomial in decreasing order of exponents;
- pull the "–" out of the bracket to start with a positive term.

a) $3x^2 + 15xa - 72a^2 =$

b) $48 - 2p^2 - 20p =$

c) $x^4 + 13x^2y^3 + 42y^6 =$

d) $x^2(x-3) - 2x(x-3) + x - 3 =$

4. **Factoring Trinomials with the Leading Coefficient ≠ 1** (decomposition method)

$$\underbrace{\boldsymbol{a}}_{mn} x^2 + \underbrace{\boldsymbol{b}}_{(mq+np)} x + \underbrace{\boldsymbol{c}}_{pq} = (mx + p)(nx + q)$$

Guessing method:

Example 2: Factor.

a) $10x^2 - 3x - 1 = (\quad x + \quad)(\quad x - \quad)$

b) $-2 + 3a^2 - 5a =$

Decomposition method:

Example 3: Factor.

a) $8x^2 + 2x - 15 =$ state the outside **product *ac***: $-8 \cdot 15 = -2 \cdot 2 \cdot 2 \cdot 3 \cdot 5$

state the **sum(difference) *b***: 2

find the **decomposition numbers**: $12, -10$

$\underbrace{8x^2 + 12x} - \underbrace{10x - 15} =$ (group 2 – 2)

$4x(\qquad) - 5(\qquad) =$ (factor GCF from each group)

$(4x - 5)(\qquad)$ (factor out the common bracket)

b) $3x^2 + 19x + 20 =$

P = 60
S = 19
# = 15, 4

c) $20p^4 - 23p^2q^2 + 6q^4 =$

P = 120
S = -23
# = $-$......, $-$.......

Remark!

- Use the **guessing method** if at least one of the outside coefficients is **prime** or **1**.
- Use the **decomposition method** if both of the outside coefficients are **composite**.

Example 3: Factor completely.

a) $4(a+b)^2 - 19(a+b) - 5 =$ (*hint:* let $x = a + b$, and factor the related polynomial $4x^2 - 19x - 5$)

b) $8(x-3)^2 - 64(x-3) + 128 =$

5. **Special Factoring**

Difference of Squares:	$\boldsymbol{a^2 - b^2 = (a+b)(a-b)}$

Example 4: Factor.

a) $x^2 - 121 =$

b) $25a^2 - 64b^2 =$

c) $x^4 - 625 =$

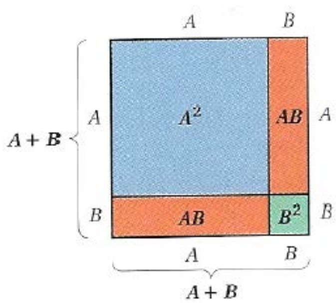
d) $20x^4y - 125y^3 =$

e) $(x+y)^2 - 100 =$

f) $49y^2 - (x-3)^2 =$

<u>*Attention:*</u> **Sum of squares is NOT factorable**. Ex. $x^2 + 9$ is prime.

Perfect Square:	$\boldsymbol{a^2 + 2ab + b^2 = (a+b)^2}$ $\boldsymbol{a^2 - 2ab + b^2 = (a-b)^2}$



Example 5: Factor.

a) $2x^2 - 28x + 98 =$

b) $16a^2 + 8ab + b^2 =$

c) $81n^2 + 144nm + 64m^2 =$

d) $-9x^3y^2 + 30x^2y - 25x =$

e) $(x-y)^2 - 2(x-y) + 1 =$

f) $x^2 + 16x + 64 - 16y^2 =$

g) $25 - a^2 + 6ab - 9b^2 =$

Sum or Difference of Cubes:	$\boldsymbol{a^3 + b^3 = (a+b)(a^2 - ab + b^2)}$ $\boldsymbol{a^3 - b^3 = (a-b)(a^2 + ab + b^2)}$

Example 6: Factor.

a) $x^3 + 1 =$

b) $8 - n^3 =$

c) $64a^3 - 27b^3 =$

d) $-x^3 - y^6 =$

e) $(x+1)^3 + 125 =$

f) $x^6 - y^9 =$

Example 7: Factor completely.

a) $y^5 + y^4 - y - 1 =$

b) $5x^3 - 5x^2y - 5xy^2 + 5y^3 =$

c) $4a^2 - 4a + 1 - b^2 + 6b - 9 =$