4.4 Properties of Logarithms

Recall from the last lecture: $\mathbf{\log_b 1 = 0}$, $\mathbf{\log_b b = 1}$, $\mathbf{\log_b b^x = x}$, $\mathbf{b^{\log_b x} = x}$

As we know, logarithms are inverses of exponential functions, so properties of logarithms arise directly from the properties of exponents, as follows:

Let $\boldsymbol{p = \log_b x}$ and $\boldsymbol{q = \log_b y}$ for $x, y \in \mathbb{R}_+$.

This is equivalent to $\boldsymbol{x = b^p}$ and $\boldsymbol{y = b^q}$.

Property:	Exponential justification:
$\mathbf{\log_b(xy) = \log_b x + \log_b y}$	$xy = b^p b^q = b^{p+q}$ so $\log_b xy = \log_b b^{p+q} = p + q$
$\mathbf{\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y}$	
$\mathbf{\log_b x^n = n \log_b x}$	$x^n = b^{pn}$ so $\log_b x^n = np$

Attention! $\log_b(x \pm y) \neq \log_b x \pm \log_b y$

Example 1: Express $\log\frac{2\sqrt{y}}{x^4}$ in terms of logarithms of $\boldsymbol{x}$, and $\boldsymbol{y}$.

Example 2: Rewrite $5\ln x + 2\ln(x+2) - \frac{1}{2}\ln(x-1)$ as a single logarithm.

Example 3: Solve $2^x = 10$,

a) applying $\log_2$:

b) applying log :

Because the exponential function is one-to-one, we must have: $\log_2 10 = \frac{\log 10}{\log 2}$

and generally we have: **The Change of Base Formula**: $\boxed{\log_b a = \frac{\log_c a}{\log_c b}}$

Example 4: Evaluate each logarithm. Round the answers to 3 decimal places.

a) $\log_5 10$ b) $\log_{\sqrt{2}} e$ c) $\log_\pi \frac{1}{2}$

Example 5: Evaluate without a calculator.

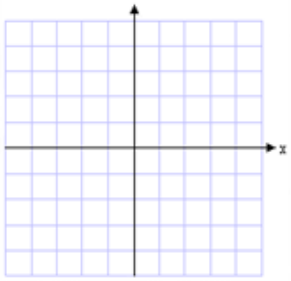
a) $\log_2 3 \cdot \log_3 4 \cdot \ldots \cdot \log_{15} 16$

b) $\frac{\log_5 0.5 + \log_{25} 4}{\log_5 2}$

c) $e^{-2 \ln 3}$

Practice 1: Using the change of base formula, prove that $\boxed{\log_{a^n} x = \frac{1}{n} \log_a x}$

Practice 2: Graph $f(x) = \log_5 (x+2)^2$ with the aid of a graphing calculator.



Richter scale: (develop to measure and compare very large or very small numbers)
The magnitude M of an earthquake with an intensity I is measured in reference to a zero-level earthquake of intensity I_0, according to the formula: $M = \log\left(\frac{I}{I_0}\right)$

Example 6: How many times as great was the intensity of the 1960 earthquake in Chile, which measured 9.5 on the Richter scale, than the San Francisco earthquake of 1906, which measured 8.3 on the Richter scale?