7.1 Rational Expressions and Functions; Multiplication and Division of Rational Expressions

A **rational expression** or **algebraic fraction** is a **quotient of polynomials**, for exp.:

$$\frac{1}{x}, \qquad \frac{x-5}{5-x}, \qquad \frac{x^2-5x+6}{x-3}, \qquad \frac{x}{x^3-9x}, \qquad x-2$$

Domain: ....

(the set of all $\boldsymbol{x}$-values that can be taken to evaluate the expression)

Definition 1: A rational function is a function defined by

$$\boldsymbol{f(x) = \frac{P(x)}{Q(x)}},$$

where $P(x)$ and $Q(x)$ are any polynomials, and $Q(x) \neq 0$.

The **domain** of such a function consists of all real numbers except for those that will make the denominator $Q(x)$ equal to 0. So $\boldsymbol{D} = \mathbb{R} \setminus \{\boldsymbol{x} | \boldsymbol{Q(x) = 0}\}$

Example 1: Find the domain.

a) $f(x) = \frac{3x}{x-2}$ b) $g(x) = \frac{x-3}{x^2-4x+3}$

Rational expressions can be **simplified by factoring** both numerator and denominator and **reducing** the **GCF**, for example:

$$\frac{x^2-5x+6}{x-3} = \underline{\qquad\qquad} = x-2, \qquad\qquad \frac{x-5}{5-x} =$$

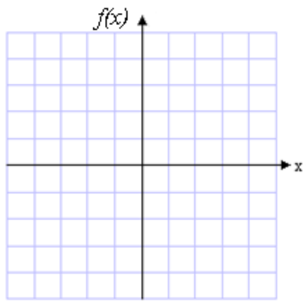
$$\frac{4x^2-100}{2x^4+250x} =$$

Notice: $\frac{-x}{y} = \frac{x}{-y} = -\frac{x}{y}$, also, opposite to $x-y$ is $y-x$ (not $x+y$), so $\frac{x-y}{y-x} = \frac{x-y}{-(x-y)} = -1$, however $\frac{x-y}{x+y}$ can't be reduced

Remember: We can **reduce only common factors**! So $\frac{x+5}{5} \neq x$ and $\frac{x+5}{5} \neq x+1$

Definition 2: Two expressions are **equivalent** in the **common domain** iff (if and only if) they produce the same values for every input from the domain.

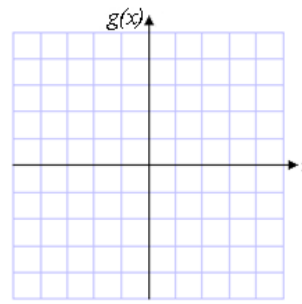
Example 2: Graph $f(x) = \frac{x^2-5x+6}{x-3}$ and $g(x) = x - 2$.



Is f the same as g?

What is the common domain?

Are the two functions the same in the common domain?



To **multiply rational expressions**, **factor** each numerator and denominator, **reduce GCF**, and then multiply according to the rule $\frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$. Leave the numerator and denominator in a factored form.

To **divide**, **multiply by** the **reciprocal** according to the rule $\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c}$.

Example 3: Perform operations.

a) $\frac{y^2-10y+9}{4y^2-4} \cdot \frac{2y+8}{y^2-5y-36} =$

b) $\frac{12x^9}{4y^2} \cdot \frac{9x^3}{32y^5} =$

c) $\frac{a^2+4a}{a^2-16} \div \frac{a^2+8a+15}{a^2+a-20} =$

d) $\frac{x^3+y^3}{2x+2y} \div \frac{x^2+y^2}{2x-2y} =$