4.5 Exponential and Logarithmic Equations

Recall: Exponential and logarithmic functions are **one-to-one**, so we can use the following properties:

If $\boldsymbol{b^x = b^y}$, then $\boldsymbol{x = y}$, and if $\boldsymbol{\log_b x = \log_b y}$, then $\boldsymbol{x = y}$.

Example 1: Solve.

a) $2^{3x-5} = 16$

b) $3^{4x} \cdot 3^{x^2} = \frac{1}{27}$

c) $8^x = 16^{3x+9}$

d) $\log_2(7 - 6x) = 5$

e) $3^{x+2} = 5^{x-1}$

f) $\log_4(x + 3) - \log_4(x - 5) = 2$

g) $3^x - 8 = -15 \cdot 3^{-x}$

h) $\ln(\ln x) = 5$

i) $\ln x^2 = (\ln x)^2$

j) $\frac{e^x+e^{-x}}{e^x-e^{-x}} = 3$

Attention! Make sure that you check the possible solutions against the domain!

Example 2: Solve. $\ln(3x) - \ln 4 = \ln(x+1)$

Some equations can't be solved algebraically. Then we approximate the solution using a graphing calculator.

Example 3: Use a graphing calculator to approximate solutions of the given equations.

a) $2^x = x^2$

b) $\ln(x+1) = 2x - 3$

Example 4: A yeast culture grows according to the equation $\boldsymbol{P} = \frac{\mathbf{50{,}000}}{\mathbf{1+250}\boldsymbol{e}^{-0.305t}}$, where $\boldsymbol{P}$ is the number of yeast and $\boldsymbol{t}$ is time in hours.

a) Graph $\boldsymbol{P}(\boldsymbol{t})$ for $\boldsymbol{t} \geq \mathbf{0}$.

b) Estimate the number of hours before the yeast population reaches 35,000.

c) Estimate the horizontal asymptote.