

## 4.6 In-class Practice

- Given the initial amount of radioactive substance  $y_0$  and the amount remaining after time  $t$ , find the exact value of  $k$  if the radioactive process of decay is modelled by the equation  $y = y_0 e^{kt}$ .
  - $y_0 = 30$  g; after 6 hr, 10 g remain
  - $y_0 = 8.1$  kg; after 4 yr, 0.9 kg remains
  - $y_0 = 10$  mg; the half-life is 100 days
  - $y_0 = 20$  mg; the half-life is 24 years

- Population Decline** A midwestern city finds its residents moving to the suburbs. Its population is declining according to the function defined by

$$P(t) = P_0 e^{-0.04t},$$

where  $t$  is time measured in years and  $P_0$  is the population at time  $t = 0$ . Assume that  $P_0 = 1,000,000$ .

- Find the population at time  $t = 1$ .
  - Estimate the time it will take for the population to decline to 750,000.
  - How long will it take for the population to decline to half the initial number?
- Cooling Soup** A hot bowl of soup is served at a dinner party. It starts to cool according to Newton's Law of Cooling, so its temperature at time  $t$  is given by

$$T(t) = 65 + 145e^{-0.05t}$$

where  $t$  is measured in minutes and  $T$  is measured in  $^{\circ}\text{F}$ .

- What is the initial temperature of the soup?
  - What is the temperature after 10 min?
  - After how long will the temperature be  $100^{\circ}\text{F}$ ?
- Heart Disease** As age increases, so does the likelihood of coronary heart disease (CHD). The fraction of people  $x$  years old with some CHD is modeled by

$$f(x) = \frac{0.9}{1 + 271e^{-0.122x}}.$$

(Source: Hosmer, D., and S. Lemeshow, *Applied Logistic Regression*, John Wiley and Sons.)

- Evaluate  $f(25)$  and  $f(65)$ . Interpret the results.
  - At what age does this likelihood equal 50%?
- Radioactive Radon** After 3 days a sample of radon-222 has decayed to 58% of its original amount.
    - What is the half-life of radon-222?
    - How long will it take the sample to decay to 20% of its original amount?

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6. **Bacteria Culture** The count in a culture of bacteria was 400 after 2 hours and 25,600 after 6 hours.
- What is the relative rate of growth of the bacteria population? Express your answer as a percentage.
  - What was the initial size of the culture?
  - Find a function that models the number of bacteria  $n(t)$  after  $t$  hours.
  - Find the number of bacteria after 4.5 hours.
  - When will the number of bacteria be 50,000?

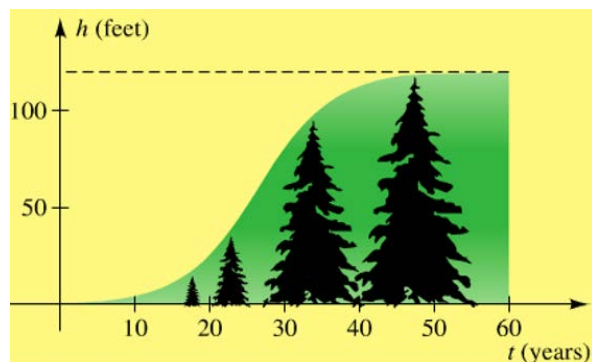
(Note: Here the relative rate of growth means the same as our exponential growth rate.)

7. **Height of trees** The growth in height of trees is frequently described by a logistic equation. Suppose the height  $h$  (in feet) of a tree at age  $t$  (in years) is

$$h = \frac{120}{1 + 200e^{-0.2t}}$$

as illustrated by the graph in the figure.

- What is the height of the tree at age 10?
- At what age is the height 50 feet?



8. If \$12,000 is invested at an interest rate of 10% per year, find the amount of the investment at the end of 3 years for each compounding method.
- |                  |                  |
|------------------|------------------|
| (a) Semiannually | (b) Monthly      |
| (c) Daily        | (d) Continuously |