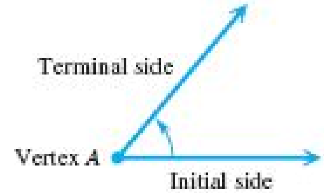
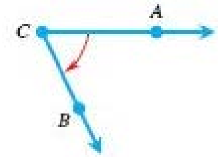
5.1 Angles and Arcs

angle – rotational space between two **rays**, called the **initial** and **terminal side**, coming from the same point, called the **vertex**; if the rotation is counterclockwise (ccw), the angle measure is positive, otherwise - negative



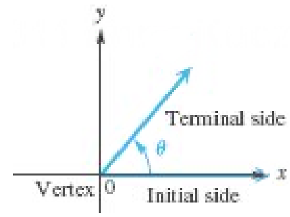
positive angle



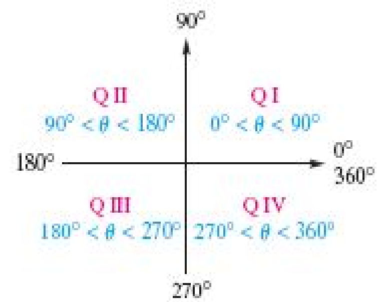
negative angle

degree – the measure of an angle formed by rotating a ray $\frac{1}{360}$ of a complete revolution;
$360°$ corresponds to a complete revolution;
$1° = 60', 1' = 60''$

complementary angles – angles that add up to $90°$
supplementary angles – angles that add up to $180°$

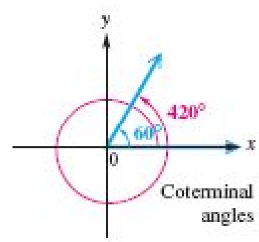


standard position – vertex at the origin and initial side on the positive *x*-axis



quadrants – four infinite regions of the Cartesian plane, bounded by two half-axes

quadrantal angles – angles in standard position with terminal side on one of the axes, such as $0°, 90°, 180°, 270°$, and so on



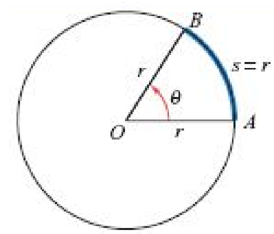
coterminal angles – angles in standard position with the same terminal side, for example $60°$ and $420°$;
A coterminal angle to $\boldsymbol{\alpha}°$ has a form $\boldsymbol{\alpha}° + \boldsymbol{n} \cdot \mathbf{360}°$ for some $n \in \mathbb{Z}$.

Example 1:
Convert angles between decimal degrees form and DMS form.

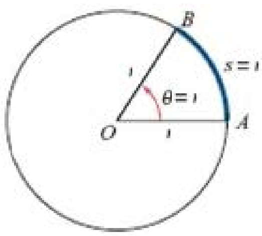
decimal degrees	**DMS**
$15.25°$	
	$65°30'45''$
$80.125°$	
	$32°10'12''$

Example 2: For each angle, find a positive coterminal angle with measure less than $360°$ and then classify the angle by quadrant.

a) $560°$ b) $820°$ c) $-75°$

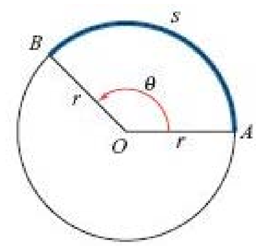


radian – the measure of the central angle subtended by an arc of length $\boldsymbol{r}$ on a circle of radius $\boldsymbol{r}$



Notice: In a **unit circle**, we have $r = 1$, so the central angle $\theta = 1\ rad$ is subtended by the arc $s = 1$.

Definition: The measure of the central angle subtended by an arc of length $\boldsymbol{s}$ in a circle of radius $\boldsymbol{r}$ is $\boxed{\boldsymbol{\theta = \frac{s}{r}}}$ radians.



What is the radian measure of

a complete revolution? ....

straight angle? ....

right angle? ....

45° ....

30° ....

60° ....

Generally, to convert from degrees to radians, multiply by $\left(\frac{\pi\ rad}{180°}\right)$.

To convert from radians to degrees, multiply by $\left(\frac{180°}{\pi\ rad}\right)$.

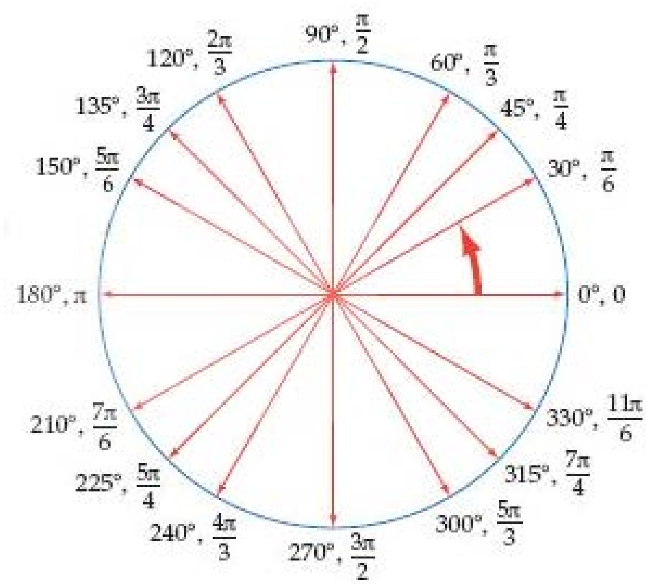
Example 2: Convert radians to degrees or degrees to radians. Give exact answer whenever possible, or round to two decimal places.

a) 75° b) 420°

c) $\frac{5\pi}{6}$ d) $-\frac{2\pi}{3}$

e) 1.37 f) 156.71°

Here is the wheel of correspondence between degree and radian measure of selected angles:



From the formula $\theta = \frac{s}{r}$, we can see that the **length s of the arc** that subtends the central angle $\boldsymbol{\theta}$ in a circle of radius $\boldsymbol{r}$ is $\boxed{\boldsymbol{s = r\theta}}$.

Remember! To use this formula, the **angle $\boldsymbol{\theta}$ must be in radians**!

Example 3:

a) Find the length of the arc $\boldsymbol{s}$ that subtends the central angle $\theta = 120°$ in a circle of radius equal to 20 cm.

b) Find the central angle subtended by the arc of length 5 cm in a circle of 4 cm in diameter.

Example 4: Find the number of radians in 2.5 revolutions.

Example 5: A pulley with radius of 10 inches uses a belt to drive a pulley with radius of 6 inches. Find the angle through which the smaller pulley turns as the 10-inch pulley makes one rotation.

Definition:

The **angular speed** of a moving point on a circular path with radius $\boldsymbol{r}$ at a constant rate of $\boldsymbol{\theta}$ radians per unit of time $\boldsymbol{t}$ is $\boxed{\boldsymbol{\omega = \frac{\theta}{t}}}$.

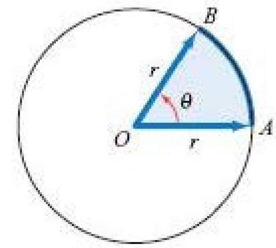
If $\boldsymbol{s}$ is the length of the arc subtending the central angle θ, then the **linear speed** of such a point is given by the formula

$$\boxed{\boldsymbol{v = \frac{s}{t} = \frac{r\theta}{t} = r\omega}}.$$

Example 6: If each tire on a car has a radius of 30 cm and the tires are rotating 400 rpm (revolutions per minute), find the speed of the car in km/h.

Example 7:

a) Find a formula for the area of a sector of a circle with radius $\boldsymbol{r}$ and central angle $\boldsymbol{\theta}$.



b) Find the area of a circle with radius of 10 cm and central angle of $\frac{2\pi}{5}$ radians.