

7.4 Rational Equations and Graphs

Guidelines of solving rational equations:

1. Determine the **domain**.
2. **Multiply by LCD.** (We can do this only for equations!)
3. **Solve** the resulting equation.
4. **Check** the possible solutions against the domain.
5. State the final **answer**.

$$\frac{4}{3x} - \frac{3}{x} = \frac{10}{3}$$

Domain =

$\neq 3x$

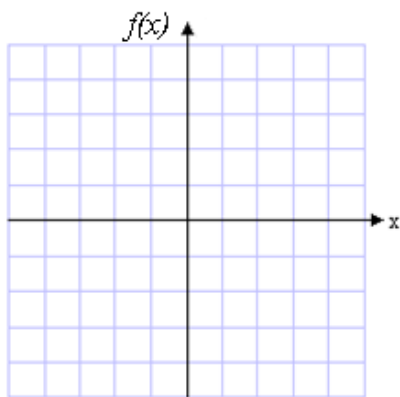
Example 1: Solve.

a) $\frac{y-1}{y-3} = \frac{2}{y-3}$

b) $\frac{x}{x-1} - \frac{2}{x+1} = \frac{4}{x^2-1}$

$$c) \quad \frac{a}{2a-6} - \frac{3}{a^2-6a+9} = \frac{a-2}{3a-9}$$

Graph and describe properties of the **reciprocal function** $f(x) = \frac{1}{x}$.



Definition 1:

If the graph of a function approaches a certain line, such a line is called an **asymptote**.

If $f(x) \rightarrow b$ when $x \rightarrow \pm\infty$, we have a **horizontal asymptote** $y = b$,

and if $f(x) \rightarrow \pm\infty$ when $x \rightarrow a$, we have a **vertical asymptote** $x = a$.

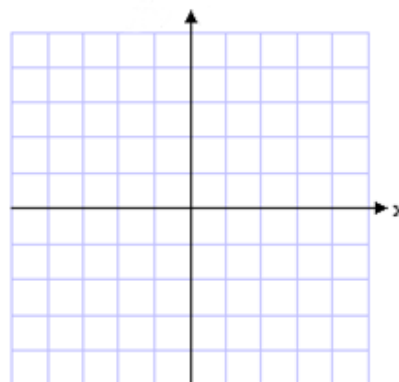
Example 2: Graph the following reciprocal functions and state their domains and asymptotes.

$$a) \quad g(x) = -\frac{3}{x}$$

Domain:

Horizontal asymptote:

Vertical asymptote:

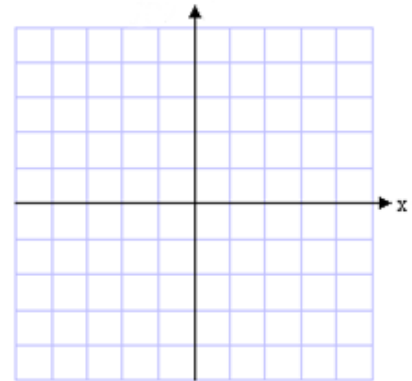


b) $h(x) = \frac{1}{x+2}$

Domain:

Horizontal asymptote:

Vertical asymptote:

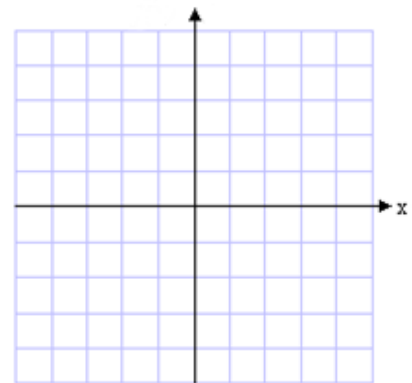


c) $k(x) = \frac{1}{x-3} + 1$

Domain:

Horizontal asymptote:

Vertical asymptote:



Example 3:

The amount of heating oil produced (in gallons per day) by an oil refinery is modeled by the rational function $f(x) = \frac{125000 - 25x}{125 + 2x}$, where x is the amount of gasoline produced (in hundreds of gallons per day). Suppose the refinery must produce 300 gal of heating oil per day to meet the needs of its customers.

- How much gasoline will be produced per day?
- Using the graph of the function, decide what happens to the amount of gasoline (x) produced as the amount of heating oil (y) produced increases.

