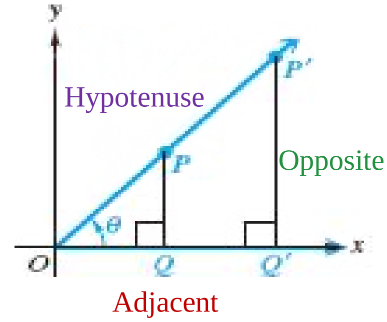
5.2 Right Triangle Trigonometry



All **right angle similar triangles** with the same acute angle $\boldsymbol{\theta}$ have the ratios between their corresponding sides equal.
These ratios depend only on the angle θ, and are called as follows:

SOH – CAH – TOA:

sine: $sin\ \theta = \frac{Opposite}{Hypotenuse}$

cosine: $cos\ \theta = \frac{Adjacent}{Hypotenuse}$

tangent: $tan\ \theta = \frac{Opposite}{Adjacent}$

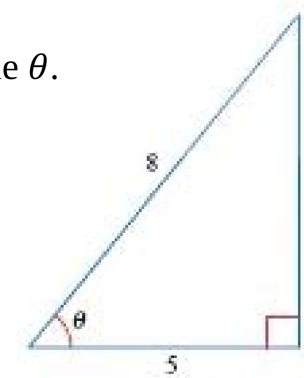
Reciprocal trig functions:

cosecant: $csc\ \theta = \frac{Hypotenuse}{Opposite} = \frac{1}{sin\ \theta}$

secant: $sec\ \theta = \frac{Hypotenuse}{Adjacent} = \frac{1}{cos\ \theta}$

cotangent: $cot\ \theta = \frac{Adjacent}{Opposite} = \frac{1}{tan\ \theta}$

Example 1: Given the triangle, find the six trigonometric ratios of angle θ. Then find angle θ in degrees and in radians.



Example 2: If θ is an acute angle of a right triangle for which $\tan\theta = \frac{3}{4}$, find $\cos\theta$, $\cot\theta$, and $\csc\theta$. (*Hint:* model the situation by drawing appropriate triangle)

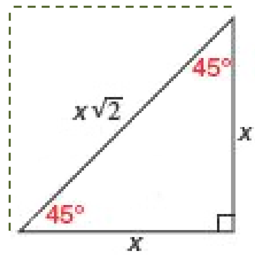
Example 3: Using a calculator, find the value (up to 4 decimal places) of

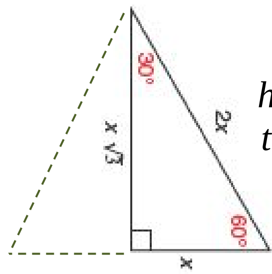
a) $\sec\frac{3\pi}{5}$ b) $\cot 35°25'$ c) $\csc 1.5$

Angles such as $30°, 45°, 60°$ occurs quite often in applications and the trig functions of such angles can be calculated exactly, using Pythagorean relations in following triangles:

Special triangles: $(45° - 45° - 90°,$ and $30° - 60° - 90°)$

half of a square

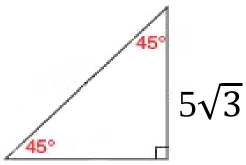




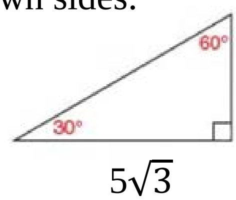
half of an equilateral triangle (Golden Triangle)

Example 4: Find the exact values of all the unknown sides:

a)

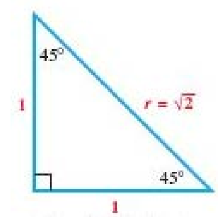


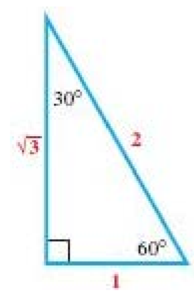
b)



The values of the six trig functions for the special angles:

$\theta°$	θ rad	$\sin\theta$	$\cos\theta$	$\tan\theta$	$\csc\theta$	$\sec\theta$	$\cot\theta$
$30°$							
$45°$							
$60°$							





Example 4: Find the exact values of the following expressions:

a) $\sin^2 30° + \cos^2 60°$

b) $\tan\frac{\pi}{3} \cdot \sec\frac{\pi}{6} - \cot\frac{\pi}{4}$

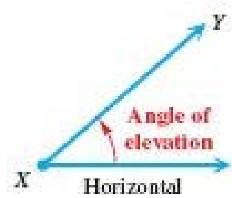
Example 5: Find the exact angle θ (in radians), if you know that θ is an acute angle and:

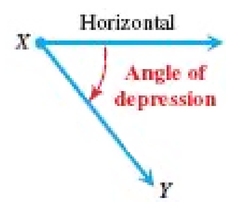
a) $\sin\theta = \frac{\sqrt{3}}{2}$

b) $\csc\theta = 2$

When solving trigonometry problems, remember to:

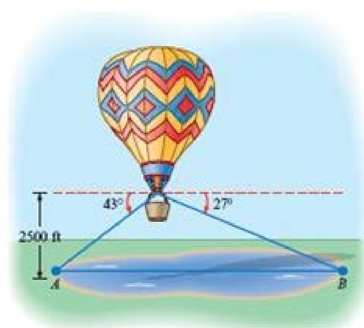
- use appropriate mode, "**degree**" or "**radians**";
- use at least 4 decimals when recording the trig functions values; ex. $\sin 32°10' =$
- use the inverse (**shift** or **2nd**) function when finding **angles**; round angles to one decimal; ex. if $\cos\theta = -.35078$, then $\theta =$
- apply **Pythagorean Theorem** <u>only</u> when working with a **right angle triangle**;
- use **exact answers** when working with **special angles**;
- an angle of **elevation** or **depression** always starts from a horizontal line:



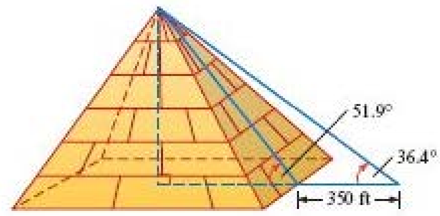


- In a triangle, the **opposite side** carries the same name as the **vertex** (vertex – capital letter; side – small letter);
- the side opposite to the **smallest angle** is the **shortest**; the side opposite to the **largest angle** is the **longest**;
- to **solve a triangle** means to find all the missing angles and sides.

Example 6: The angle of depression to one side of a lake, measured from a balloon 2500 feet above the lake, is $43°$. The angle of depression to the opposite side of the lake is $27°$. Find the width of the lake.



Example 7: The angle of elevation to the top of the Egyptian pyramid of Cheops is 36.4°, measured from a point 350 feet from the base of the pyramid. The angle of elevation from the base of a face of the pyramid is 51.9°. Find the height of the Cheops pyramid.



Example 8: Show that the area of a triangle *ABC* can be calculated by the formula $\boxed{\boldsymbol{A = \frac{1}{2}ab\sin\theta}}$, where θ is the angle between the sides a and b.