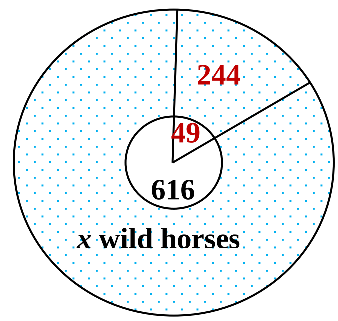
7.5 Applications of Rational Equations

Proportion Problems: $\frac{category\ I\ before}{category\ II\ before} = \frac{category\ I\ after}{category\ II\ after}$

Example 1:

To estimate the number of wild horses in California, a forest ranger catches 616 wild horses, tags them, and releases them. Later, 244 horses are caught, and it is found that 49 of them are tagged. Assuming that the horses mix freely when they are released, estimate how many wild horses are in California.

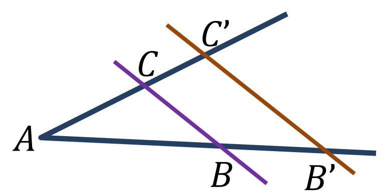
Solution: Let ***x*** be the number of wild horses in California.



	population		**sample**
tagged horses:			
	$\frac{\quad}{x}$	=	$\frac{\quad}{\quad}$
wild horses:			

Answer: There are approximately .... wild horses in California.

Thales' Theorem: Two triangles are **similar** iff their corresponding sides are **proportional**.

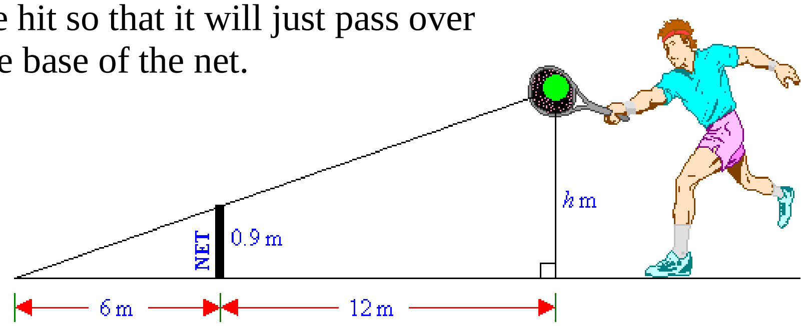


$$\triangle ABC \sim \triangle AB'C' \Leftrightarrow \frac{AB}{AB'} = \frac{AC}{AC'} = \frac{BC}{B'C'}$$

Example 2:

Using the information given in the diagram below, find the value of the height *h* (in meters) at which the tennis ball must be hit so that it will just pass over the net and land 6 metres away from the base of the net.

Solution:



Answer: The height is ....

Motion Problems:

	R ·	**T =**	**D**
movement I			
movement II			
total			

Note: In motion problems we may add <u>*times*</u> or <u>*distances*</u> but we usually <u>**do not add rates!**</u>

Example 3:

The speed of a car is 5 mph faster than the speed of a bus. The car travels 220 mi in the same time it takes the bus to travel 200 mi. Find the speed of the car and the speed of the bus.

Solution: Let ***r*** be the speed of the bus.

	R ·	**T =**	**D**
car			
bus			

Fill up the *"time column"* using the formula: $\boldsymbol{T = \frac{D}{R}}$

Answer: The speed of the bus is .... and the speed of the car is ....

Example 4:

A plane travels 100 mi against the wind in the same time that it takes to travel 120 mi with the wind. If the speed of the wind is 20 mph, find the rate of the plane in still air.

Solution: Let ***r*** be the rate of the plane in still air.

	R ·	**T =**	**D**
with the wind			
against the wind			

Answer: The rate of the plane in still air is ....

Work Problems:

Work problems can be solved similarly as motion problems:

Rate of performing the job · **Time =** amount of **Job** done (usually 1)

Example 5:

David can paint a house in 12 hours. Bill can paint the same house in 9 hours. How long would it take them to paint the house together?

Solution:

	R ·	**T** =	**Job done**
David			1
Bill			1
together			1

Note:

In work problems we usually *add rates* but **do not add times!**

Answer: It will take them .... hours, if working together.

Example 6:

A tank can be filled in 9 hr and drained in 11 hr. How long will it take to fill the tank if the drain is left open?

Solution:

	R ·	**T** =	**Job done**
filling in			1
draining out			1
together			1

Answer: It will take .... to fill the tank.

Solving Formulas for a Given Variable:

Rules to keep in mind:

- Use **opposite operations** to clear unwanted factors or addends

$\frac{A}{B} = \frac{C}{D} + 1$, solve for C

$A = \frac{(x+y)B}{2}$, solve for x

- Keep your **variable in the numerator** (by multiplying by **LCD** or by taking **reciprocals** if you deal with a proportion)

$\frac{1}{A} + \frac{1}{B} = \frac{1}{C}$, solve for C

$\frac{1}{A} + \frac{1}{B} = \frac{1}{C}$, solve for B

- Bring your **variables to one place** (by keeping them in numerators and then **factoring** out)

$P = \frac{At}{1+At}$, solve for t

$I = \frac{nE}{nr+E}$, solve for n