8.2 Rational Exponents

Notation: $\sqrt[n]{x} = x^{\frac{1}{n}}$ and generally $\sqrt[n]{x^m} = \left(\sqrt[n]{x}\right)^m = x^{\frac{m}{n}}$

The **index of the radical** becomes the **denominator of the rational exponent.**

Example 1: Change from radical to exponential form (or the other way around) and simplify if possible.

a) $\sqrt{5} =$

b) $\sqrt[3]{2} =$

c) $\sqrt[5]{3^2} =$

d) $\sqrt[4]{25} =$

e) $\sqrt{16x^4y^2} =$

f) $\left(\sqrt[3]{xy^2}\right)^5 =$

g) $x^{\frac{2}{7}} =$

h) $(3x)^{-\frac{2}{5}} =$

i) $3x^{-\frac{1}{2}} =$

j) $(x+2y)^{\frac{2}{3}} =$

Example 2: Evaluate. (*Hint:* It is helpful to change numbers into **powers of prime numbers**, if possible)

a) $8^{-\frac{1}{3}} =$

b) $-64^{\frac{2}{3}} =$

c) $(-125)^{-\frac{1}{3}} =$

d) $\left(\frac{16}{81}\right)^{-\frac{3}{4}} =$

Example 3: Simplify. Assume that all variables are positive. Leave your answer in a simplified radical form. (*Hint:* convert to rational exponents first.)

a) $\sqrt{5^{10}} =$

b) $\sqrt[6]{x^{18}} =$

c) $\sqrt[4]{y^2} =$

d) $\sqrt{a} \cdot \sqrt[3]{a^2} =$

e) $\sqrt{\sqrt[3]{2}} =$

f) $\frac{\sqrt[4]{x}}{\sqrt[5]{x}} =$

Example 4: Simplify. Assume that all variables are positive. Leave your answer with positive exponents.

a) $x^{\frac{2}{5}} \cdot x^{-\frac{3}{4}} =$

b) $\dfrac{a^{\frac{1}{3}}}{a^{-\frac{1}{4}} \cdot a^{\frac{2}{3}}} =$

c) $\left(\dfrac{p^{-\frac{1}{4}} \cdot q^{-\frac{3}{2}}}{3^{-1} \cdot p^{-2} \cdot q^{-\frac{3}{2}}}\right)^{-2} =$

d) $7y^{\frac{8}{5}}\left(y^{-\frac{8}{5}} - 3y^{-\frac{3}{5}}\right) =$

Example 5: Simplify. Assume that all variables are positive. Leave your answer in a simplified exponential form.

a) $\sqrt{\sqrt[3]{\sqrt[4]{x}}} =$

b) $\dfrac{\sqrt[4]{x^5}}{\sqrt[3]{x^2}} =$

Example 6: Find the domain of the function $f(x) = (x+1)^{\frac{1}{2}}(x-2)^{-\frac{1}{3}}$.