8.3 Simplifying Radical Expressions and the Distance Formula

Since radicals are really fractional exponents, all exponential rules apply to radicals as well.

So we have: $\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{ab}$ and $\frac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\frac{a}{b}}$

However, remember that $\sqrt[n]{a \pm b} \neq \sqrt[n]{a} \pm \sqrt[n]{b}$ so $\sqrt{a^2 \pm b^2} \neq a \pm b$

To simplify a radical with all positive variables: $\sqrt{160x^6y^{11}} =$

- **factor completely** the radicand $\sqrt{2^5 \cdot 5 \cdot x^6 \cdot y^{11}} =$
- take out of the radical appropriate power of each factor (**the quotient on exponential level**) $2^2 \cdot x^3 \cdot y^5\sqrt{2 \cdot 5 \cdot y}$

Example 1: Simplify. Assume all variables are positive.

a) $\sqrt[3]{-2^6x^9y^{21}} =$

b) $\sqrt[4]{3^5a^{14}b^7} =$

c) $-\sqrt{\frac{75}{a^8}} =$

d) $2x \cdot \sqrt[3]{\frac{64}{x^{10}}} =$

e) $\sqrt[4]{x^2} =$

f) $\sqrt[10]{32} =$

Notice: The last two examples suggest the rule: $\sqrt[kn]{a^{km}} = \sqrt[n]{a^m}$

Example 2: Multiply and simplify. Assume all variables are positive.

a) $\sqrt{8x} \cdot \sqrt{6x^3} =$

b) $\sqrt[3]{3x} \cdot \sqrt[3]{9x^4} =$

c) $\sqrt{2} \cdot \sqrt[3]{2} =$

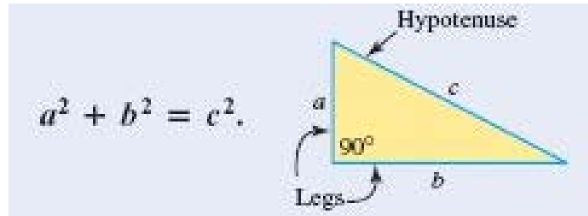
d) $\sqrt[3]{a} \cdot \sqrt[5]{a^2} =$

Why it is important to know how to simplify roots?
In many geometry problems we are seeking the exact solutions, for example:

- the exact length of a third side in a right angle triangle,
- the exact distance between two given points.

Pythagorean Theorem:

A triangle is a **right angle triangle** iff the **sum of squares of legs** (shorter sides) is equal to the **square of the hypotenuse** (the longest side).



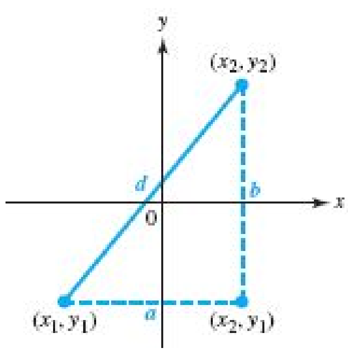
Example 3: Find the unknown side, given that:

a) $a = 4, c = 12$

b) $a = 3\sqrt{2}, b = 5\sqrt{2}$

c) $a = 1, c = \sqrt{n+1}$

To find the **distance between two points** with coordinates $(\boldsymbol{x_1}, \boldsymbol{y_1})$ and $(\boldsymbol{x_2}, \boldsymbol{y_2})$, apply Pythagorean equation to the right triangle as on the diagram:



$$d^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2$$

so $$\boldsymbol{d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}}$$

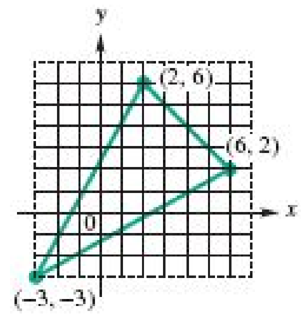
Example 4: Find the distance between each pair of points.

a) $(2, -3), (-5, 7)$

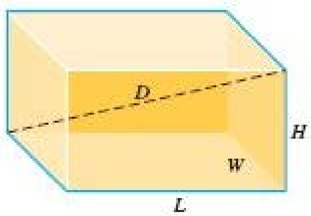
b) $(\sqrt{2}, -\sqrt{3}), (2\sqrt{2}, \sqrt{3})$

Practice:

1. Find the perimeter of the triangle.



2. Develop a formula for the diagonal ***D*** of a box with the length ***L***, width ***W***, and height ***H***.



3. Find the length of the diagonal of a cube with side 5 cm.