

**8.4, 8.5**     **Operations on Radical Expressions; Rationalizing the Denominator**

**Addition:** We can add or subtract (combine) radicals of the **same order** and with the **same radicands** by combining their coefficients - similarly as we combine like terms in polynomials.

*Example 1:* Simplify and combine.

a)  $2\sqrt{12} - 5\sqrt{48} + 4\sqrt{75} =$

b)  $9\sqrt[3]{16} + \sqrt[3]{54} - \sqrt{8} =$

c)  $\sqrt{3a} + 2\sqrt{27a^5} =$

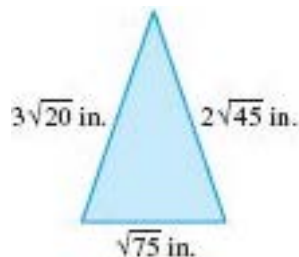
d)  $\sqrt{4x+4} - \sqrt{x^3+x^2} =$

e)  $2\sqrt{\frac{40}{9}} - 7\frac{\sqrt{3}}{\sqrt{27}} =$

f)  $\sqrt[3]{\frac{5}{a^6}} + 3a\sqrt[3]{\frac{125}{a^9}} =$

*Practice:*

1. Find the perimeter of the given triangle.



**Multiplication:** Multiply radicals the same way as polynomials. Use “foiling” process and special formulas, if possible. Simplify radicals in each product.

*Example 2:* Multiply and simplify.

a)  $2\sqrt{5}(3\sqrt{5} - 5\sqrt{2}) =$

b)  $(\sqrt{3} + 2)(\sqrt{6} - \sqrt{3}) =$

c)  $(\sqrt{x} - \sqrt[3]{2})(\sqrt{x} + \sqrt[3]{2}) =$

d)  $(2\sqrt{3} + 5\sqrt{2})(2\sqrt{3} - 5\sqrt{2}) =$

e)  $(3 + \sqrt{5})^2 =$

f)  $(2\sqrt{p} - 3\sqrt{q})^2 =$

g)  $((\sqrt{5} - \sqrt{3}) + \sqrt{6})((\sqrt{5} - \sqrt{3}) - \sqrt{6}) =$

h)  $(\sqrt{x+1} - \sqrt{x-1})^2 =$

**Rationalization of denominators:** Keep denominators free of radical expressions.

- If **one-term** denominator – first simplify it, then **multiply the numerator and denominator by the same radical**, to match the powers of the radicand to the order of the radical.

*Example 3:* Rationalize the denominator.

a)  $\frac{2}{\sqrt{2}}$

b)  $\frac{3\sqrt{5}}{2\sqrt{3}}$

c)  $-\sqrt{\frac{3}{5xy^2}}$

d)  $\sqrt[3]{\frac{5}{2x^2}}$

e)  $\frac{x}{\sqrt[4]{x^2y^3}}$

f)  $\frac{-1}{\sqrt[5]{ab^4}}$

- If **two-term** denominator – **multiply the numerator and denominator by the conjugate bracket** ( $(a + b)$  and  $(a - b)$  are conjugates) to use **difference of squares** and get rid of the radicals in the denominator.

*Example 4:* Rationalize the denominator.

a)  $\frac{\sqrt{2}}{1-\sqrt{2}}$

b)  $\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

c)  $\frac{a-16}{\sqrt{a}-4}$

d)  $\frac{2}{3\sqrt{5}+2\sqrt{3}}$

*Example 5:* Simplify.

a)  $\frac{2-6\sqrt{2}}{4}$

b)  $\frac{10x+\sqrt{400x^3}}{5x}$