## 5.7 Graphing Trigonometric Functions

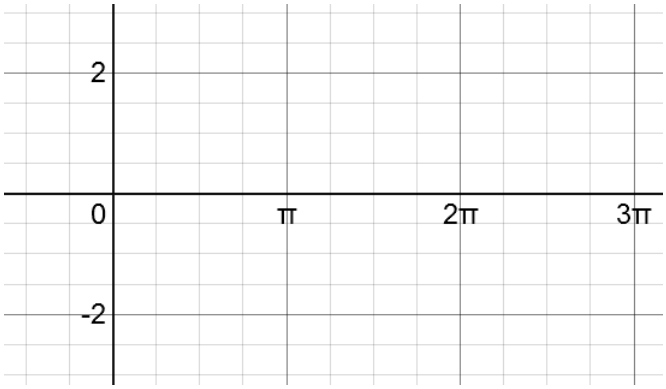
*Recall:*

* **vertical translations:**
  - to obtain $g(x) = \boldsymbol{f(x) + d}$ from $f(x)$, **translate** the graph of $f$ by a vector $\langle \boldsymbol{0, d} \rangle$

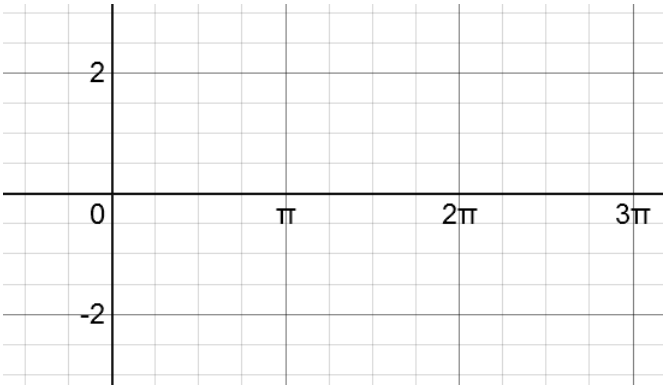
* **horizontal translations:**
  - to obtain $g(x) = \boldsymbol{f(x + c)}$ from $f(x)$, **translate** the graph of $f$ by a vector $\langle \boldsymbol{-c, 0} \rangle$

*Example 1:* Graph on the same grid.

a) $f(x) = \sin x$ and $g(x) = \sin x + 1$




b) $f(x) = \cos x$ and $g(x) = \cos\left(x - \frac{\pi}{4}\right)$




When graphing trigonometric functions,

- **vertical translation** by a vector $\langle \boldsymbol{0, d} \rangle$ corresponds to a vertical shift of the **midline** ($y = 0$) of the basic graph. So the new midline is at $y = \boldsymbol{d}$;
- **horizontal translation** by a vector $\langle \boldsymbol{c, 0} \rangle$ is customary called a **phase shift** by $\boldsymbol{c}$.

*In summary:*

To graph one period of $\boxed{\boldsymbol{f(x) = A(trig\ function)(Bx + C) + D}}$, with $B > 0$,

- keep in mind the **basic shape** of the trig function, including the sign;
- to see the **phase shift** explicitly, rewrite the function to the form

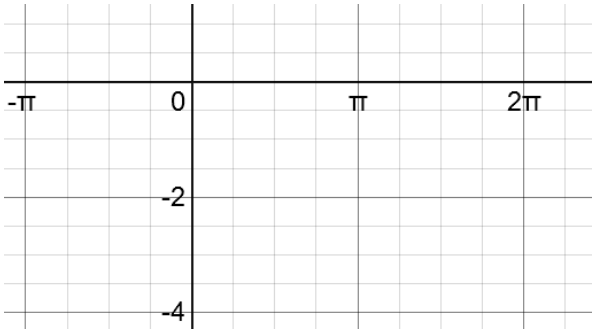
$$\boldsymbol{f(x) = A(trig\ function)\left(B\left(x + \frac{C}{B}\right)\right) + D}$$

- sketch dashed vertical lines at $\boldsymbol{x = -\frac{C}{B}}$ (**phase shift**) and $\boldsymbol{x = -\frac{C}{B} + P}$, where $\boldsymbol{P = \frac{period\ of\ f}{B}}$;
- sketch dashed horizontal lines at $\boldsymbol{y = D}$ **(midline)**, $\boldsymbol{y = D + |A|}$, and $\boldsymbol{y = D - |A|}$;
- subdivide the segment on the midline into 4 equal sections;
- plot the key values of the basic trig function within the obtained grid (rectangle) and join them, following the shape of the function.

*Example 2:* State the shape, amplitude, period, midline, and the phase shift. Then graph the function by sketching appropriate rectangle for one full period and marking the function key values first.

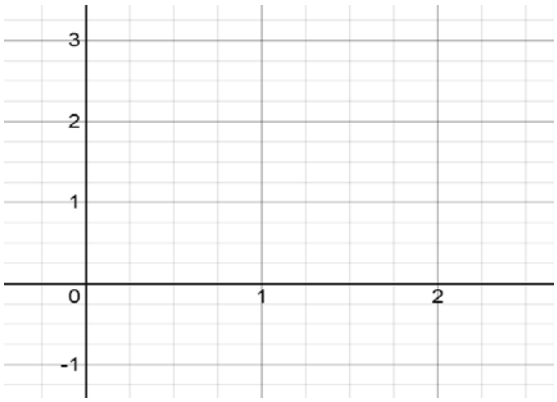
a) $f(x) = -\frac{1}{2}\sin\left(2x + \frac{\pi}{2}\right) - 3$

shape:
amplitude:
period:
midline:
phase shift:



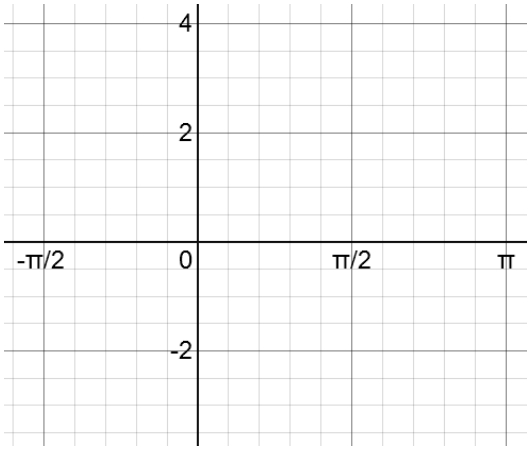

b) $f(x) = 2\cos\left(-\pi x + \frac{\pi}{2}\right) + 1$

shape:
amplitude:
period:
midline:
phase shift:



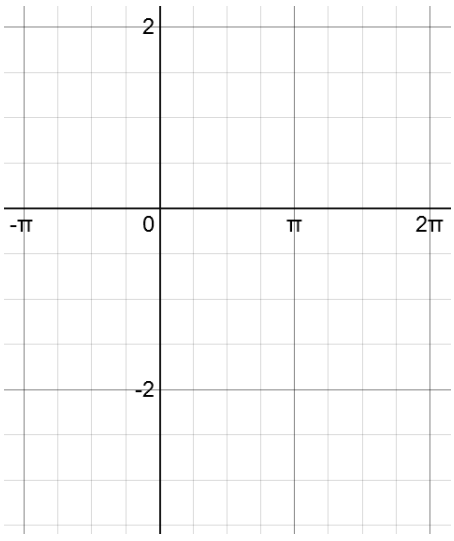

c) $f(x) = \frac{1}{2}\tan\left(\frac{5}{2}x - \pi\right) + \frac{3}{4}$

shape:
amplitude:
period:
asymptotes:
midline:
phase shift:



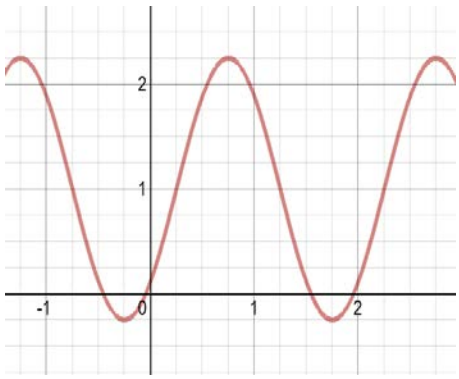

b) $f(x) = -\frac{3}{2}\csc\left(2x + \frac{\pi}{2}\right) - 1$

shape:
amplitude:
period:
asymptotes:
midline:
phase shift:

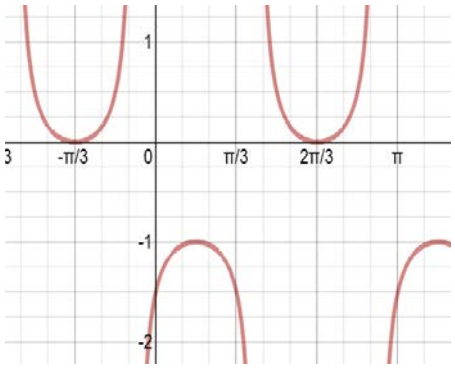


*Example 3:* Find a formula for the function given by the graph.

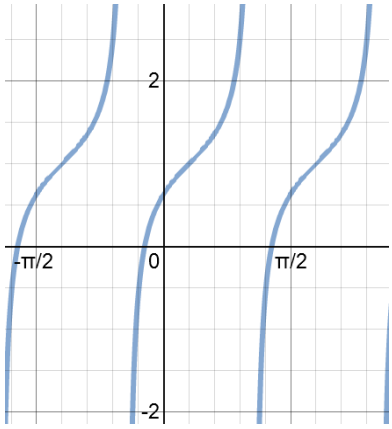
a)



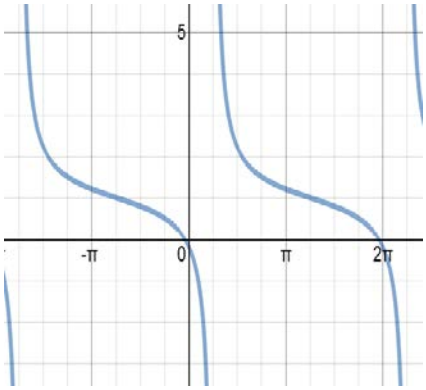

b)




c)



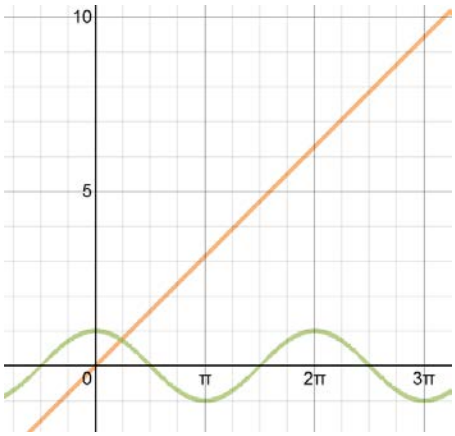

d)

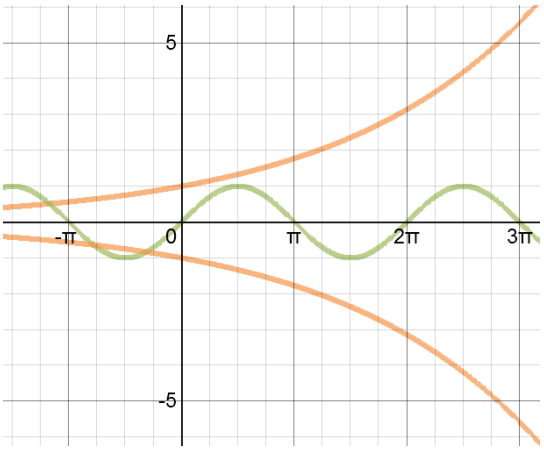



*Example 6:* Graph.

a) $f(x) = x + \cos x$

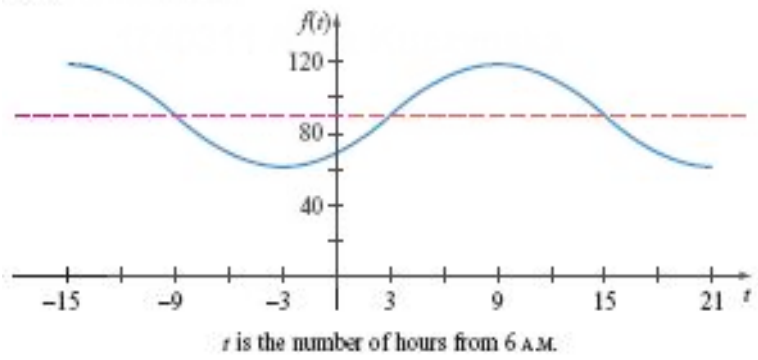
b) $g(x) = 2^x \sin x$



*Example 7:* Find an equation of a cotangent function with period $\frac{\pi}{2}$ and phase shift $-\frac{\pi}{4}$.

*Example 8:* During a summer day, the ground temperature at a desert location was recorded and graphed as a function of time.




*t* is the number of hours from 6 A.M.

The graph can be approximated by
$f(t) = A\cos(Bt + C) + D$, where *t* is the number of hours since 6 a.m.
Find the equation of the graph and approximate the temperature at 1:00 p.m.