

8.6 Radical Equations

A radical equation is an equation containing radical signs, for example:

$$\sqrt{x} = 4$$

$$3 - \sqrt{2x} = 1$$

$$\sqrt{2x + 3} = -1$$

$$\sqrt[3]{x} = x$$

General strategy of solving radical equations:

1. **Isolate one radical** on one side of the equation.
2. **Square** (or raise to n^{th} power if needed) both sides of the equation. Remember to correctly apply **perfect square formula** $(a \pm b)^2 = a^2 \pm 2ab + b^2$ if needed.
3. If you still have a radical, repeat the first two steps until all radicals are cleared.
4. **Solve** as linear or quadratic equation.
5. **Check if the possible solutions work for the original equation.** If NOT – drop them. The ones that do not work are called **extraneous** solutions.
6. State the final **answer**.

Example 1:

$$\sqrt{5x - 1} + 5 = x$$

$$\sqrt{5x - 1} = x - 5$$

$$5x - 1 = x^2 - 10x + 25$$

$$x^2 - 15x + 26 = 0$$

$$(x - 13)(x - 2) = 0$$

$$x = 13 \text{ or } x = 2$$

Check:

if $x = 13$: if $x = 2$:

Answer: $x = 13$

Example 2: Solve.

a) $\sqrt{5 - x} = 2\sqrt{-x}$

b) $\sqrt[3]{2x - 6} = \sqrt[3]{4 - 3x}$

c) $\sqrt{x + 1} - \sqrt{x - 1} = 2$

d) $(3x + 4)^{\frac{1}{2}} - (2x - 4)^{\frac{1}{2}} = 2$

Example 3: Solve for the indicated variable.

a) $T = 2\pi\sqrt{\frac{L}{32}}$ for L

b) $N = \frac{1}{2\pi}\sqrt{\frac{a}{r}}$ for r