6.1 Fundamental Trigonometric Identities and Their Applications

*Recall the Fundamental Trig **Identities** (statements true for any real input):*

Reciprocal identities	$\sin x = \frac{1}{\csc x}$	$\cos x = \frac{1}{\sec x}$	$\tan x = \frac{1}{\cot x}$
Ratio identities	$\tan x = \frac{\sin x}{\cos x}$	$\cot x = \frac{\cos x}{\sin x}$	
Pythagorean identities	$\sin^2 x + \cos^2 x = 1$	$\tan^2 x + 1 = \sec^2 x$	$1 + \cot^2 x = \csc^2 x$
Odd–even identities	$\sin(-x) = -\sin x$ $\cos(-x) = \cos x$	$\tan(-x) = -\tan x$ $\cot(-x) = -\cot x$	$\sec(-x) = \sec x$ $\csc(-x) = -\csc x$

We can use the above identities to **verify** (prove) some other identities.

To check if an equation is an **identity**, we can either

- find an input value that will make the equation not true (**disprove** it);
- or **transform equivalently** the left (or/and possibly the right) side of the equation to match the other side of the equation, employing some of the following techniques:
  - performing indicated operations (ex. adding fractions; squaring or factoring)
  - using previously proven identities
  - rewriting in terms of a single trig function
  - rewriting in terms of sine and cosine only
  - multiplying the numerator and denominator by a conjugate
  - simplifying keeping in mind the goal that we want to achieve

Example 1: Show that the given equation is <u>not</u> an identity.

a) $\sqrt{\cos^2 x} = \cos x$ b) $\ln\left(\frac{1}{\sin x}\right) = \frac{1}{\ln \sin x}$

Example 2: Verify the identity.

a) $\cos^2 x - \sin^2 x = 2\cos^2 x - 1$ b) $\frac{\sin x}{1-\cos x} = \csc x + \cot x$

c) $\frac{1}{\sin^2 x}+\frac{1}{\cos^2 x}=\csc^2 x \sec^2 x$

d) $\frac{\sin x+\tan x}{1+\cos x}=\tan x$

e) $\ln|\csc x-\cot x|=-\ln|\csc x+\cot x|$

f) $\frac{1-\sin x+\cos x}{1+\sin x+\cos x}=\frac{\cos x}{\sin x+1}$ (multiply by $(\sin x+1)$)