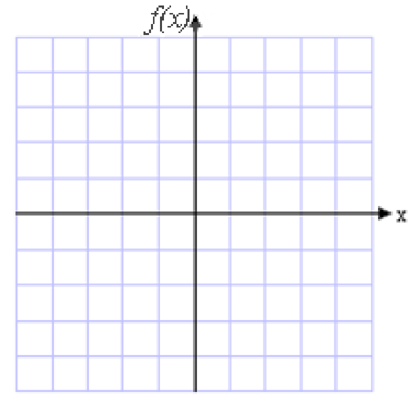
9.1-9.3 Quadratic Equations, Completing the Square, and Quadratic Formula

Quadratic equation – any equation that can be written in the form $\boldsymbol{ax^2+bx+c=0}$, where $a, b, c \in \mathbb{R}$, $a \neq 0$. This is called **standard** (or **general**) **form**.

Different ways of solving a quadratic equation:

Method:	*Example:*
Graphing	To find solutions of $x^2-4=0$, find the $\boldsymbol{x}$-intercepts of the function $f(x)=x^2-4$.  solution set =
Factoring	$x^2-4=0$ $(x+2)(x-2)=0$, so the solution set is $\{\boldsymbol{-2,2}\}$
Square Root Property	$x^2-4=0$ $x^2=4$ $\sqrt{x^2}=4$ $\lvert x\rvert=2$, so the solution set is $\{\boldsymbol{-2,2}\}$
Completing the Square	$x^2+2x-5=0$ $x^2+2x=5$ $x^2+2x+1=5+1$ (complete the square by **adding perfect square of half of the middle coefficient** to both sides of the equation) $(x+1)^2-1=5$ (or equivalently take **half of the middle coefficient**, **square it**, and **subtract perfect square of what you took**) $(x+1)^2=6$ (solve by square root property) so the solution set is $\left\{\boldsymbol{-1\pm\sqrt{6}}\right\}$
Quadratic Formula $\boldsymbol{x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}}$	$3x^2+2x-5=0$ use quadratic formula with $\boldsymbol{a}=3, \boldsymbol{b}=2, \boldsymbol{c}=-5$ $x_{1,2}=\dfrac{-2\pm\sqrt{2^2-4\cdot 3(-5)}}{2\cdot 3}=$ so $x=1$ or $x=-\frac{5}{3}$

Example 1: Solve by completing the square.

a) $(2x+5)^2 = 10$

b) $x^2 - 7x = 10$

c) $r^2 + \frac{2}{5}r = \frac{4}{5}$

d) $2x^2 + 3x - 1 = 0$

Quadratic Formula $\boldsymbol{x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}}$ arises from solving the standard form of quadratic equation $\boldsymbol{ax^2 + bx + c = 0}$ in the process of **completing the square**:

Notice 1: The number of solutions of a quadratic equation depends on the expression $\boldsymbol{b^2 - 4ac}$, called the **discriminant** and often denoted $\boldsymbol{\Delta}$:
if $\boldsymbol{\Delta} > 0$, then we have **two** different **real solutions**;
if $\boldsymbol{\Delta = 0}$, then we have **one** double **real solution**;
if $\boldsymbol{\Delta} < 0$, then we have two different nonreal (complex) solutions;

In terms of the discriminant $\boldsymbol{\Delta = b^2 - 4ac}$,
the Quadratic Formula takes the form: $\boldsymbol{x = \frac{-b \pm \sqrt{\Delta}}{2a}}$

Notice 2: If the **discriminant** is a **perfect square**, then the equation can be solved by **factoring**; otherwise, we need to use the quadratic formula.

Example 2: Solve by using the Quadratic Formula.

a) $3p^2 = -8p - 1$

b) $-2t(t+2) = -3$

c) $\frac{1}{y} + \frac{1}{y+2} = \frac{1}{3}$

d) $2x = \sqrt{11x + 3}$

Example 3: Using the discriminant, tell the number and type of solutions for each equation without actually solving it. Can the equation be solved by factoring?

a) $5x^2 + 8x + 3 = 0$

b) $x^2 + x + 1 = 0$

c) $2y^2 + 2y - 3 = 0$

d) $4x^2 - 28x + 49 = 0$

Equations reducible to quadratic:

Example 4: Solve.

a) $6m^4 - 19m^2 + 15 = 0$

b) $3x^{-2} - x^{-1} - 14 = 0$

c) $(x-4)^2 + (x-4) - 20 = 0$

d) $2 + \frac{5}{3x-1} = \frac{-2}{(3x-1)^2}$

Example 5:

The Hudson River flows at a rate of 3 mph. A patrol boat travels 60 mi upriver and returns in a total time of 9 hr. What is the speed of the boat in still water?

Example 6:

It takes 4 hr for two carpet installers to install a carpet in a room. If each installer works alone, one of them could do the job in 1 hr less time than the other. How long would it take each carpet installer to complete the job alone?