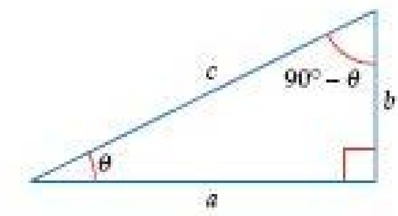
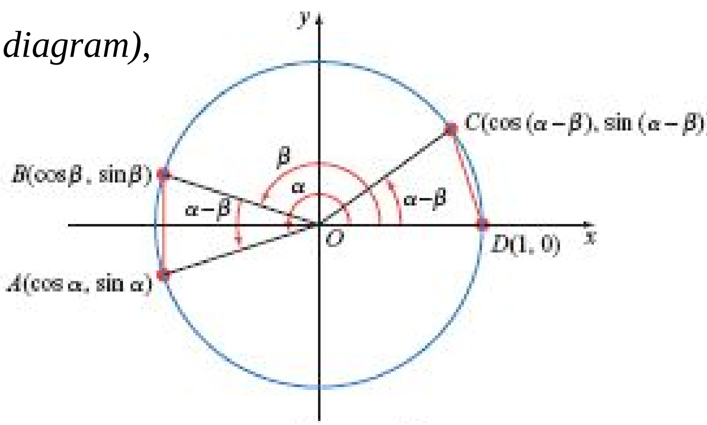
6.2 Sum, Difference, and Cofunction Identities

Cofunction Identities:

$\sin(90° - \theta) = \cos\theta$	$\cos(90° - \theta) = \sin\theta$
$\tan(90° - \theta) = \cot\theta$	$\cot(90° - \theta) = \tan\theta$
$\sec(90° - \theta) = \csc\theta$	$\csc(90° - \theta) = \sec\theta$



By equating the distances *AB* and *CD (as on the diagram),* we can obtain the following identity:
(proof in your textbook)



$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

Now, by replacing β by $-\beta$, we have

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

From previous identities, we have

$$\sin(\alpha + \beta) = \cos(90° - \alpha - \beta) = \cos(90° - \alpha)\cos\beta + \sin(90° - \alpha)\sin\beta$$
$$= \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

And again, replacing β by $-\beta$, we have

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

Using identities for sine and cosine of sum of angles, we have

$$\tan(\alpha + \beta) = \frac{\sin(\alpha+\beta)}{\cos(\alpha+\beta)} = \frac{\sin\alpha\cos\beta + \cos\alpha\sin\beta}{\cos\alpha\cos\beta - \sin\alpha\sin\beta} = \frac{\frac{\sin\alpha\cos\beta}{\cos\alpha\cos\beta} + \frac{\cos\alpha\sin\beta}{\cos\alpha\cos\beta}}{\frac{\cos\alpha\cos\beta}{\cos\alpha\cos\beta} - \frac{\sin\alpha\sin\beta}{\cos\alpha\cos\beta}}$$
$$= \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

Replacing β by $-\beta$, we have

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

Example 1: Find the exact value of $\sin\left(\frac{\pi}{4} - \frac{\pi}{3}\right)$.

$$\cos(\alpha \pm \beta) = \cos\alpha \cos\beta \mp \sin\alpha \sin\beta$$
$$\sin(\alpha \pm \beta) = \sin\alpha \cos\beta \pm \cos\alpha \sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha \tan\beta}$$

Example 2: Find the exact value of

a) $\tan 75°$

b) $\cos 15°$

Example 3: Evaluate the expression.

a) $\sin\frac{x}{5}\cos\frac{4x}{5} + \cos\frac{x}{5}\sin\frac{4x}{5}$

b) $\dfrac{\tan 3x - \tan 4x}{1 + \tan 3x \tan 4x}$

Example 4: Given $\sin\alpha = \frac{24}{25}$, α is in QII, $\tan\beta = \frac{3}{4}$, and β is in QIII, find

a) $\cos(\beta - \alpha)$

b) $\sin(\alpha + \beta)$

c) $\tan(\alpha + \beta)$

Example 5: Rewrite the expression in terms of trig functions of angle θ.

a) $\cos(\theta + \pi)$

b) $\sin(\theta - (2k + 1)\pi)$

Example 6: Verify identities.

$$\cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$
$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$$
$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

a) $\boxed{\sin 2\theta = 2\sin\theta\cos\theta}$

b) $\boxed{\cos 2\theta = \cos^2\theta - \sin^2\theta}$

c) $\boxed{\tan 2\theta = \frac{2\tan\theta}{1 - \tan^2\theta}}$

d) $\frac{\cos 4\theta}{\sin\theta} - \frac{\sin 4\theta}{\cos\theta} = \frac{\cos 5\theta}{\sin\theta\cos\theta}$

e) $\sin 3x = 3\sin x - 4\sin^3 x$