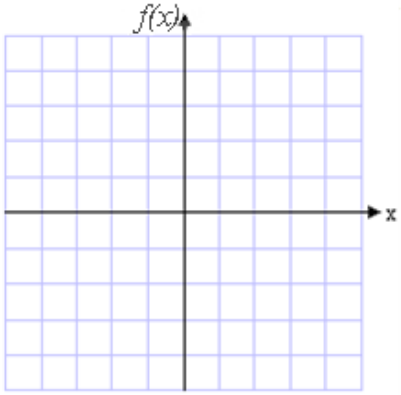
## 9.5 Properties and Graphs of Quadratic Function

**quadratic function** – any function that can be written in the form $\boldsymbol{f(x) = ax^2 + bx + c}$, where $a, b, c \in \mathbb{R}$, $a \neq 0$; this is the **standard** (or **general**) **form** of a **parabola**

Observe properties of the graph of the **basic parabola** $\boldsymbol{f(x) = x^2}$.

| $x$ | $f(x)$ |
|---|---|
| $-2$ | |
| $-1$ | |
| $0$ | |
| $1$ | |
| $2$ | |

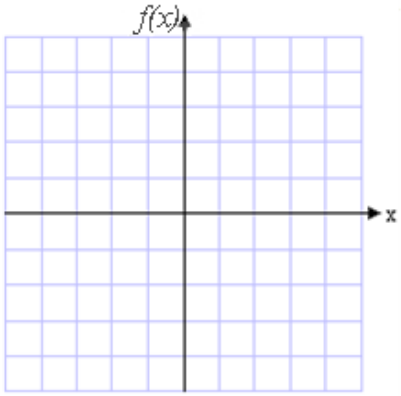



shape
opens
domain =
range =
vertex =
axis of symmetry:

Now, observe how those properties change when we modify the function to

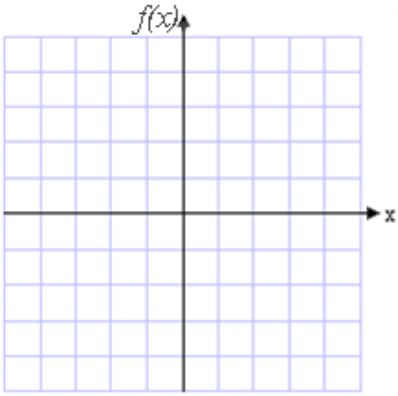
$f(x) = x^2 + 1$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | |
| $-1$ | |
| $0$ | |
| $1$ | |
| $2$ | |



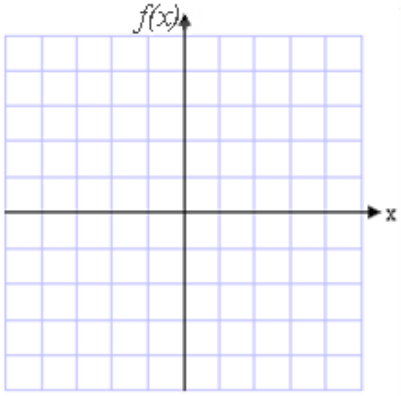

$f(x) = x^2 - 2$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | |
| $-1$ | |
| $0$ | |
| $1$ | |
| $2$ | |



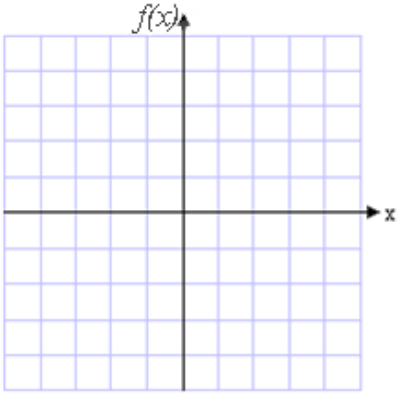

$f(x) = (x + 1)^2$

| $x$ | $f(x)$ |
|---|---|
| $-3$ | |
| $-2$ | |
| $-1$ | |
| $0$ | |
| $1$ | |



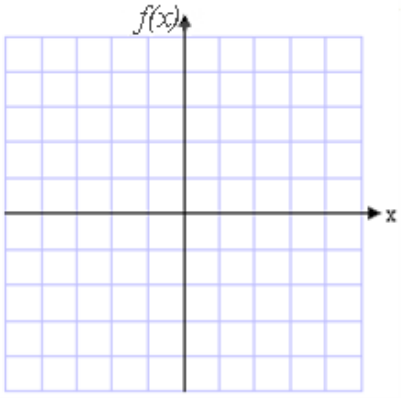

$f(x) = (x - 2)^2$

| $x$ | $f(x)$ |
|---|---|
| $0$ | |
| $1$ | |
| $2$ | |
| $3$ | |
| $4$ | |



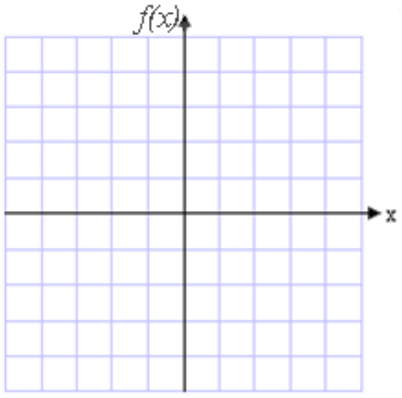
$f(x) = 2x^2$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | |
| $-1$ | |
| 0 | |
| 1 | |
| 2 | |



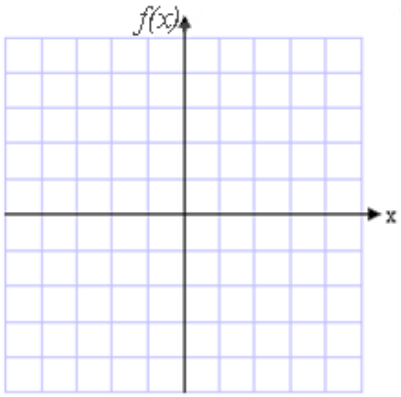

$f(x) = -\frac{1}{2}x^2$

| $x$ | $f(x)$ |
|---|---|
| $-2$ | |
| $-1$ | |
| 0 | |
| 1 | |
| 2 | |




$f(x) = -(x-2)^2 + 1$

| $x$ | $f(x)$ |
|---|---|
| 0 | |
| 1 | |
| 2 | |
| 3 | |
| 4 | |




shape
opens
domain =
range =
vertex =
axis of symmetry:

*Notice:*
By completing the square, we can always convert the **standard form** of a quadratic function $(\boldsymbol{x}) = \boldsymbol{ax^2 + bx + c}$ $(a, b, c \in \mathbb{R},\ a \neq 0)$ to the **vertex form** $\boldsymbol{f(x) = a(x-h)^2 + k}$.

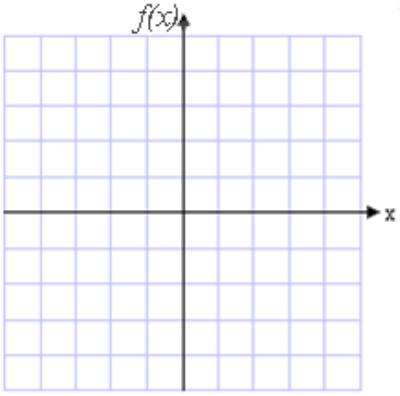
*Conclusion:* The graph of $\boldsymbol{f(x) = a(x-h)^2 + k}$ has following properties:

- the shape of **parabola**, similar to $f(x) = \boldsymbol{x^2}$;
- **opens up** if $\boldsymbol{a} > 0$ and **opens down** if $\boldsymbol{a} < 0$;
- the **vertex** at $(\boldsymbol{h}, \boldsymbol{k})$;
- **axis of symmetry**: vertical line $\boldsymbol{x = h}$
- the **domain** of such parabola $= \mathbb{R}$
- the **range** $= \begin{cases} [k,\infty), & if\ a > 0 \\ (-\infty, k], & if\ a < 0 \end{cases}$
- if $\boldsymbol{a} > 0$, $\boldsymbol{k}$ is the **minimum** value; if $\boldsymbol{a} < 0$, $\boldsymbol{k}$ is the **maximum** value

*Practice 1:* Identify the **vertex**, **axis of symmetry**, **opening**, and **range**. Then, **graph** the function and observe how the a-value influences the shape.

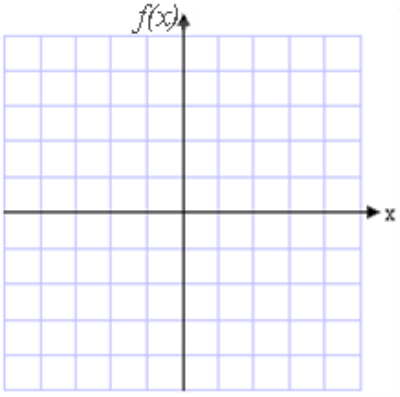
a) $f(x) = \frac{1}{2}(x-3)^2 + 1$

vertex:
axis of symmetry:
opening:
range:
shape:



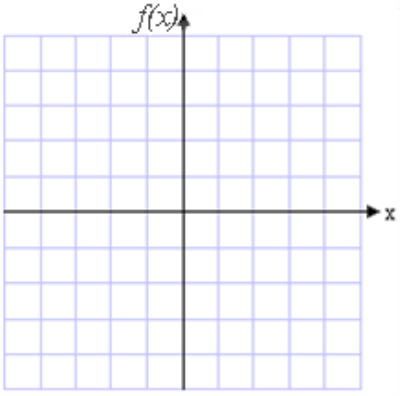

b) $f(x) = 2(x+3)^2 - 2$

vertex:
axis of symmetry:
opening:
range:
shape:




c) $f(x) = -\frac{1}{3}(x+1)^2 + 2$

vertex:
axis of symmetry:
opening:
range:
shape:




d) $f(x) = -3(x-2)^2 + 4$

vertex:
axis of symmetry:
opening:
range:
shape: