1. (a) To define the inverse sine function, we restrict the domain of sine to the interval ____________. On this interval the sine function is one-to-one, and its inverse function $\sin^{-1}$ is defined by $\sin^{-1} x = y \Leftrightarrow \sin$ _____ = _____. For example, $\sin^{-1}\frac{1}{2} =$ _____ because sin _____ = _____.

(b) To define the inverse cosine function we restrict the domain of cosine to the interval ____________. On this interval the cosine function is one-to-one and its inverse function $\cos^{-1}$ is defined by $\cos^{-1} x = y \Leftrightarrow$ cos _____ = _____. For example, $\cos^{-1}\frac{1}{2} =$ _____ because cos _____ = _____.

2. The cancellation property $\sin^{-1}(\sin x) = x$ is valid for x in the interval ____________. Which of the following is not true?

(a) $\sin^{-1}\left(\sin\frac{\pi}{3}\right) = \frac{\pi}{3}$

(b) $\sin^{-1}\left(\sin\frac{10\pi}{3}\right) = \frac{10\pi}{3}$

3. Find the exact value of each expression, if it is defined.

a) $\sin^{-1} 1$ b) $\sin^{-1}\frac{\sqrt{3}}{2}$ c) $\sin^{-1} 2$

d) $\sin^{-1}(-1)$ d) $\sin^{-1}\frac{\sqrt{2}}{2}$ e) $\sin^{-1}(-2)$

f) $\cos^{-1}(-1)$ g) $\cos^{-1}\frac{1}{2}$ h) $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$

i) $\cos^{-1}\left(\frac{\sqrt{2}}{2}\right)$ j) $\cos^{-1} 1$ k) $\cos^{-1}\left(-\frac{\sqrt{2}}{2}\right)$

l) $\tan^{-1}(-1)$ m) $\tan^{-1}\sqrt{3}$ n) $\tan^{-1}\frac{\sqrt{3}}{3}$

o) $\tan^{-1} 0$ p) $\tan^{-1}(-\sqrt{3})$ r) $\tan^{-1}\left(-\frac{\sqrt{3}}{3}\right)$

s) $\cot^{-1}\frac{\sqrt{3}}{3}$ t) $\csc^{-1}(-2)$ u) $\sec^{-1}(-2)$

4. Use a calculator to find an approximate value of each expression correct to five decimal places, if it is defined.

a) $\sin^{-1}\left(-\frac{8}{9}\right)$ b) $\cos^{-1}\frac{4}{9}$

c) $\tan^{-1}(-26)$ d) $\cot^{-1} 10$

e) $\sec^{-1}(-5.118)$ f) $\csc^{-1} 1.25$

5. **Two Different Compositions** Let f and g be the functions

$$f(x) = \sin(\sin^{-1} x)$$

and

$$g(x) = \sin^{-1}(\sin x)$$

By the cancellation properties, $f(x) = x$ and $g(x) = x$ for suitable values of x. But these functions are not the same for all x. Graph both f and g to show how the functions differ. (Think carefully about the domain and range of $\sin^{-1}$).

6. Find the exact value of the expression, if it is defined.

a) $\sin\left(\sin^{-1}\frac{1}{4}\right)$

b) $\tan\left(\tan^{-1}\frac{3}{2}\right)$

c) $\cos^{-1}\left(\cos\frac{5\pi}{6}\right)$

d) $\tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right)$

e) $\sin^{-1}\left(\sin\frac{5\pi}{6}\right)$

f) $\cos^{-1}\left(\cos\left(-\frac{\pi}{6}\right)\right)$

g) $\cos^{-1}\left(\cos\frac{17\pi}{6}\right)$

h) $\tan^{-1}\left(\tan\frac{4\pi}{3}\right)$

i) $\sin^{-1}\left(\sin\frac{11\pi}{4}\right)$

j) $\tan^{-1}\left(\tan\frac{2\pi}{3}\right)$

k) $\tan\left(\sin^{-1}\frac{1}{2}\right)$

l) $\cos(\sin^{-1}0)$

m) $\tan\left(\sin^{-1}\frac{\sqrt{2}}{2}\right)$

n) $\cos\left(\sin^{-1}\frac{\sqrt{3}}{2}\right)$

o) $\sin\left(\tan^{-1}(-\sqrt{3})\right)$

p) $\cos\left(\tan^{-1}(-1)\right)$

r) $\cos\left(2\sin^{-1}\frac{1}{4}\right)$

s) $\sin\left(2\cos^{-1}\frac{1}{5}\right)$

7. Write ech expression as an algebraic (nontrigonometric) expression in terms of u, for $u > 0$.

a) $\sin(\arccos u)$

b) $\tan(\arccos u)$

c) $\cot(\arcsin u)$

d) $\cos(\arcsin u)$

e) $\sin\left(2\sec^{-1}\frac{u}{2}\right)$

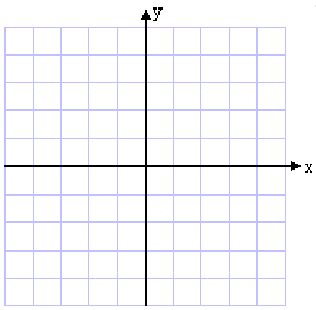
f) $\cos\left(2\tan^{-1}\frac{3}{u}\right)$

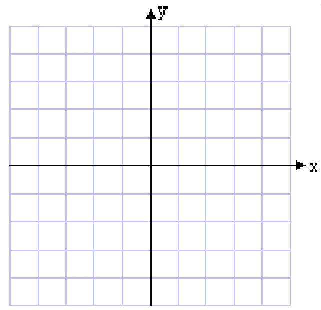
g) $\tan\left(\sin^{-1}\frac{u}{\sqrt{u^2+2}}\right)$

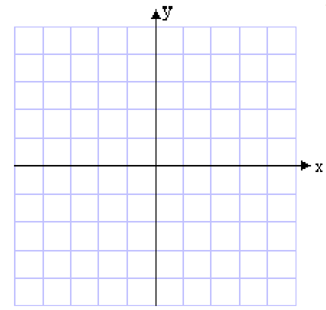
h) $\sec\left(\cos^{-1}\frac{u}{\sqrt{u^2+5}}\right)$

8. Sketch the graph of the equation.

a) $y = \sin^{-1}(x+1)$ b) $y = \cos^{-1}\frac{1}{2}x$ c) $y = 2 + \tan^{-1}x$







9. Given the equation $y = f(x)$ of the function f, find the domain and range of f and then solve the equation for x in terms of y.

a) $y = \frac{1}{2}\sin^{-1}(x-3)$ b) $y = 3\tan^{-1}(2x+1)$

10. ***Landscaping Formula*** A shrub is planted in a 100-ft-wide space between buildings measuring 75 ft and 150 ft tall. The location of the shrub determines how much sun it receives each day. Show that if θ is the angle in the figure and x is the distance of the shrub from the taller building, then the value of θ (in radians) is given by

$$\theta = \pi - \arctan\left(\frac{75}{100-x}\right) - \arctan\left(\frac{150}{x}\right).$$