9.6 Applications of Properties of Quadratic Function

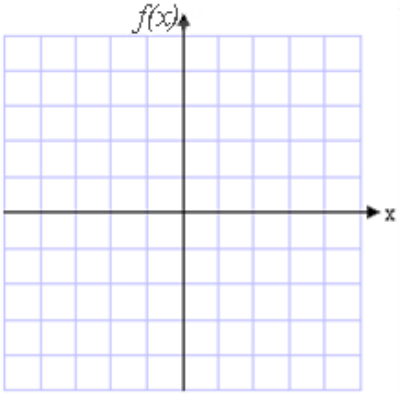
Vertex form of a quadratic function $\boldsymbol{f(x) = a(x-h)^2 + k}$ carries a lot of information about the graph, such as vertex, axis of symmetry, opening, range, **extreme values** (**maximum** or **minimum**). This will be very useful in solving some **optimization problems**.

How to find the vertex of a quadratic function given in standard form?
$\boldsymbol{f(x) = ax^2 + bx + c}$

Conclusion: The **vertex** of a parabola $f(x) = \boldsymbol{a}x^2 + \boldsymbol{b}x + \boldsymbol{c}$ (with $a \neq 0$) has coordinates $\left(\frac{\boldsymbol{-b}}{\boldsymbol{2a}}, \frac{\boldsymbol{-(b^2-4ac)}}{\boldsymbol{4a}}\right)$ or equivalently $\left(\frac{\boldsymbol{-b}}{\boldsymbol{2a}}, \frac{\boldsymbol{-\Delta}}{\boldsymbol{4a}}\right)$, or equivalently $\left(\frac{-b}{2a}, f\left(\frac{\boldsymbol{-b}}{\boldsymbol{2a}}\right)\right)$.

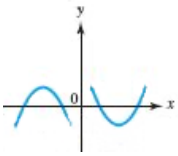
Therefore, the **line of symmetry** is $\boldsymbol{x = \frac{-b}{2a}}$.

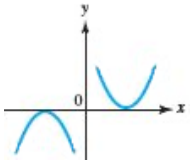
Example 1: Find the discriminant, vertex, shape, opening, and *x*-intercepts of the parabola $f(x) = 2x^2 + 8x + 5$. Then give the vertex form of the parabola and graph it.



Recall: The number of *x*-intercepts of a quadratic function depends on the **discriminant** $\boldsymbol{\Delta = b^2 - 4ac}$,

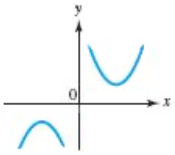
if $\boldsymbol{\Delta} > 0$, then we have **two** *x*-intercepts $x_{1,2} = \frac{-b \pm \sqrt{\boldsymbol{\Delta}}}{2a}$;





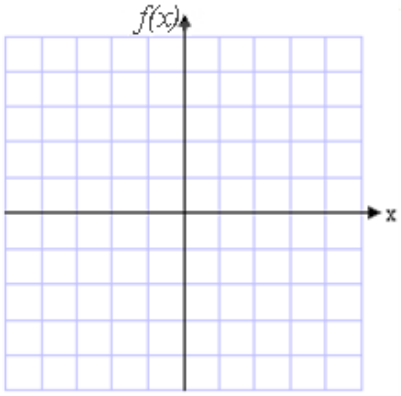
if $\boldsymbol{\Delta} = \mathbf{0}$, then we have **one** x-intercept $x = \frac{-b}{2a}$;

if $\boldsymbol{\Delta} < 0$, then we don't have any x-intercept;

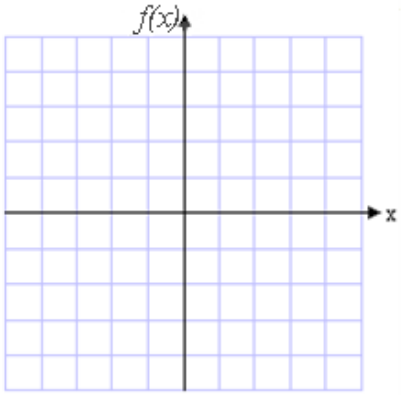


Example 2: For each function, state the number of *x*-intercepts, the shape, opening, and the vertex. Then graph it, and state the extreme value.

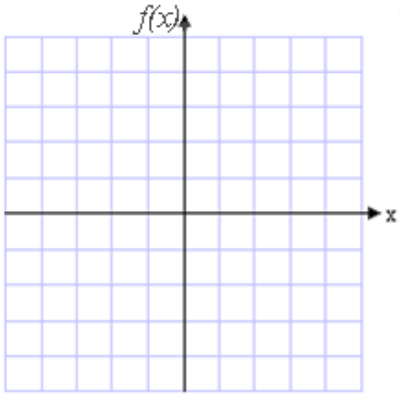
a) $f(x) = x^2 - x + 1$



b) $f(x) = -x^2 + 4x - 4$



c) $f(x) = \frac{1}{2}x^2 + 2x - 1$

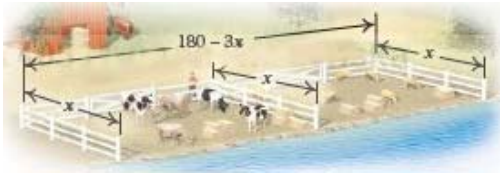


Example 3:

What is the minimum product of two numbers whose difference is 6? What are the numbers?

Example 4:

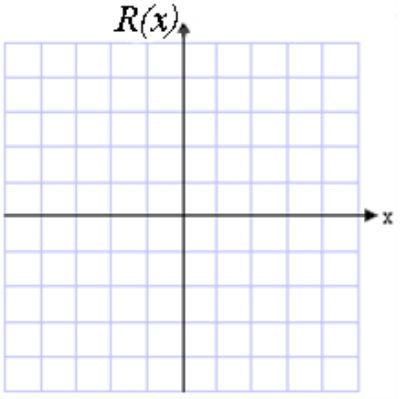
A rancher needs to enclose two adjacent rectangular corrals, one for sheep and one for cattle. If a river forms one side of the corrals and 180 yd of fencing is available, what is the largest total area that can be enclosed?



Example 5:

For a trip to a resort, a charter bus company charges a fare of \$48 per person, plus \$2 per person for each unsold seat on the bus. If the bus has 42 seats and x represents the number of unsold seats, find the following:

a) a function $R(x)$ that describes the total revenue from the trip (*Hint:* the total number of people riding times the price per ticket)



b) the graph of the function from part a)

c) the number of unsold seats that produces the maximum revenue

d) the maximum revenue