9.7 <u>Polynomial and Rational Inequalities</u>

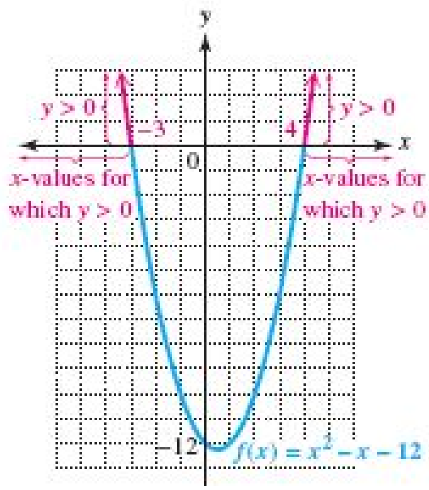
Examples of polynomial inequalities: $x^2 - x \geq 12$, $x(x-2)(x+1) < 0$

Examples of rational inequalities: $\frac{x}{x+1} > 0$, $\frac{x-1}{x+2} \leq 3$, $\frac{(x+3)^2}{x^4} > -1$

Such inequalities can be solved graphically or algebraically.

Example 1: Solve the inequality $x^2 - x \geq 12$ using the graph of the related function $f(x) = x^2 - x - 12$.

Solution:



We can see from the graph of $f(x) = x^2 - x - 12$ that $f(x) \geq 0$ for $\in$ ....

Therefore, $x^2 - x - 12 \geq 0$ and equivalently $x^2 - x \geq 12$ for $x \in (-\infty, -3] \cup [4, \infty)$.

Example 2: Solve $x(x-2)(x+1) < 0$ by analyzing signs of all factors.

Solution:

x				
$x-2$				
$x+1$				
result	-1	0	2	

$(x - 2 > 0 \text{ for } x > 2)$

$(x + 1 > 0 \text{ for } x > -1)$

So $x(x-2)(x+1) < 0$ for $x \in$ ....

Example 3: Solve $\frac{(x+3)^2}{x^4} > -1$.

Solution:

Notice that $\frac{(x+3)^2}{x^4} \geq 0$ and $0 > -1$, so $\frac{(x+3)^2}{x^4} > -1$ is true for all real numbers in its domain. Therefore, the solution set is $\mathbb{R} \setminus \{0\}$.

Example 4: Solve $\frac{x-1}{x+2} \leq 3.$

Solution ***(general strategy):***

1. Keep **one side** equal to **0**.

2. Keep the other side in the form of a **single fraction**.

3. **Factor completely** both, the numerator and denominator.
4. **Analyze signs of all factors** and record it in a sign table, on a number line.

5. Read the solution set from the table of signs. **Watch the endpoints** and decide which one to include in the solution set. Exclude any values that are not in the domain of the original inequality.

Practice:

1. Solve each inequality and graph the solution set. Give the answer in interval notation.

a) $3x^2 - 6x + 2 \leq 0$

b) $\frac{x}{x+2} > 2x$