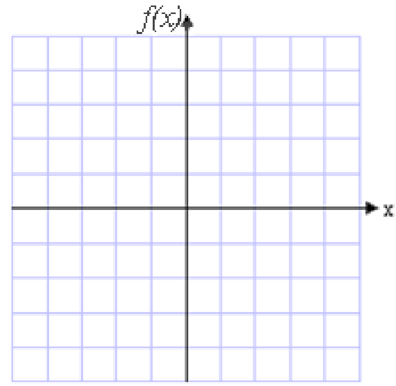
11.1 Properties of Graphs of Various Functions and their Translations

Observe the basic shapes and properties of graphs of the commonly used functions:

absolute value function $f(x) = |x|$

x	$f(x)$
-2	
-1	
0	
1	
2	

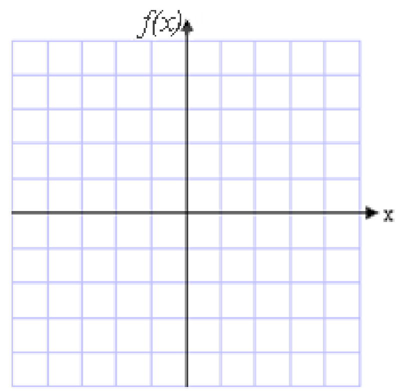
domain:
range:
symmetry:
vertex:



root function $f(x) = \sqrt{x}$

x	$f(x)$
0	
$\frac{1}{4}$	
1	
4	

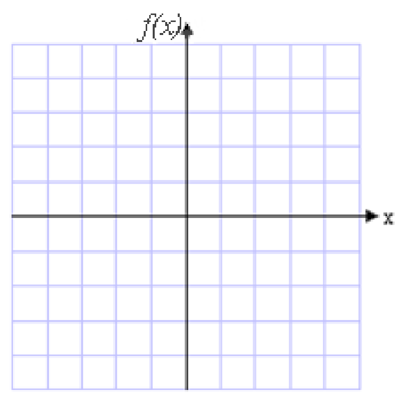
domain:
range:
vertex:



reciprocal function $f(x) = \frac{1}{x}$

x	$f(x)$
-2	
-1	
0	
$\frac{1}{2}$	
1	
2	

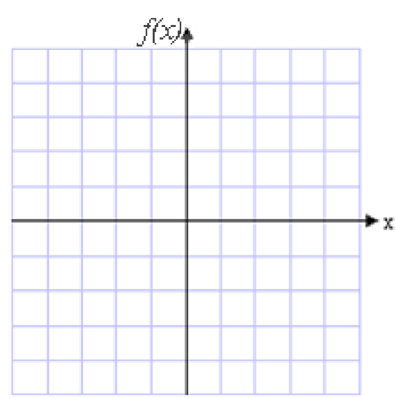
domain:
range:
symmetry:
asymptotes:



cubic function $f(x) = x^3$

x	$f(x)$
-2	
-1	
0	
1	
2	

domain:
range:
symmetry:



greatest integer function $f(x) = [\![x]\!] = \{n \ \ for\ x \in [n, n+1)$

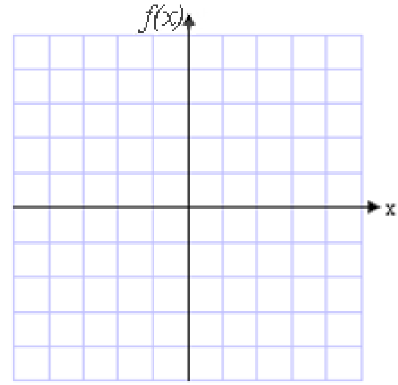
(step function)

x	$f(x)$
-0.5	
0	
0.5	
0.99	
1	
2.3	

domain:

range:

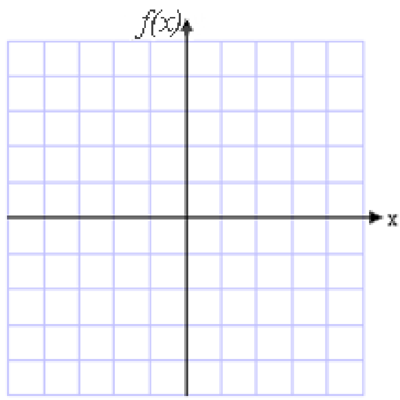
jumps:



Now, observe how those properties change when we modify the function to

$f(x) = |x - 1|$

x	$f(x)$
-2	
-1	
0	
1	
2	

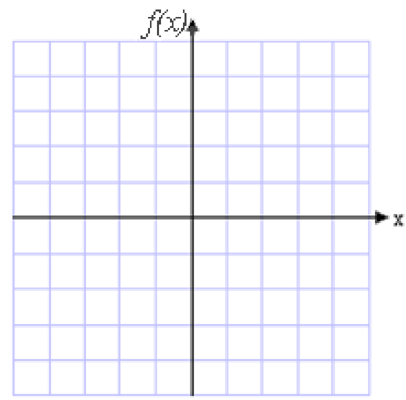


vertex:

range:

$f(x) = |x - 1| - 2$

x	$f(x)$
-2	
-1	
0	
1	
2	

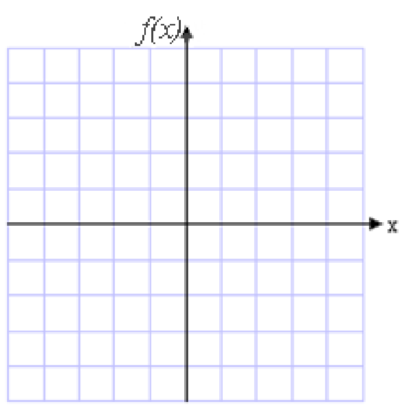


vertex:

range:

$f(x) = \frac{1}{x+1}$

x	$f(x)$
-3	
-2	
-1	
0	
1	



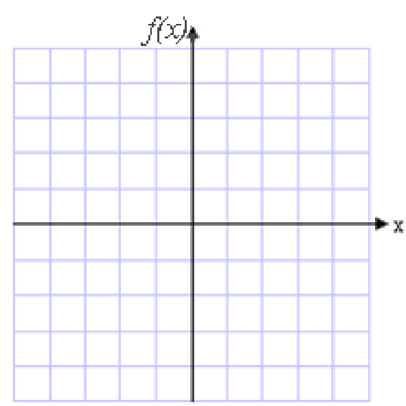
domain:

range:

asymptotes:

$f(x) = \frac{1}{x+1} - 2$

x	$f(x)$
-1	
0	
2	
3	
4	



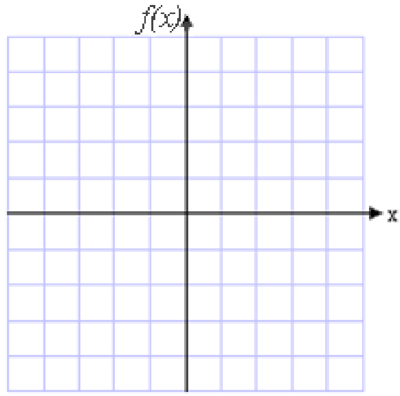
domain:

range:

asymptotes:

$f(x) = \sqrt{x+3}$

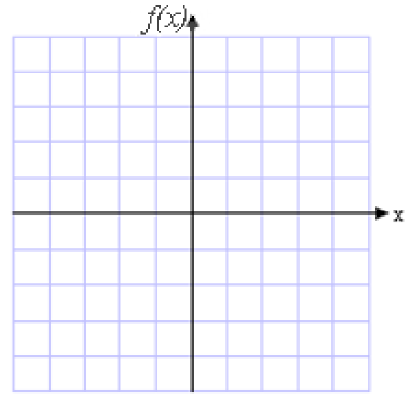
x	$f(x)$
-2	
-1	
0	
1	
2	



domain:
range:
vertex:

$f(x) = \sqrt{x+3} + 1$

x	$f(x)$
-2	
-1	
0	
1	
2	



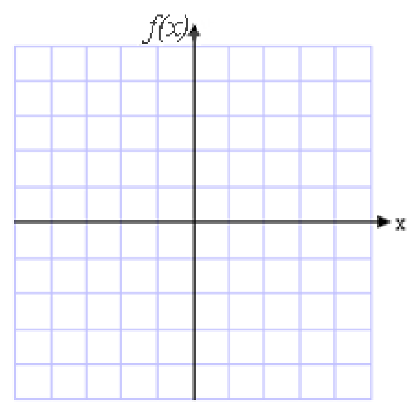
domain:
range:
vertex:

Observation: To graph a function $\boldsymbol{f(x-a)+b}$, **translate** the graph of $\boldsymbol{f(x)}$ **by the vector** $\langle \boldsymbol{a}, \boldsymbol{b} \rangle$ (that means $\boldsymbol{a}$ steps horizontally and $\boldsymbol{b}$ steps vertically).

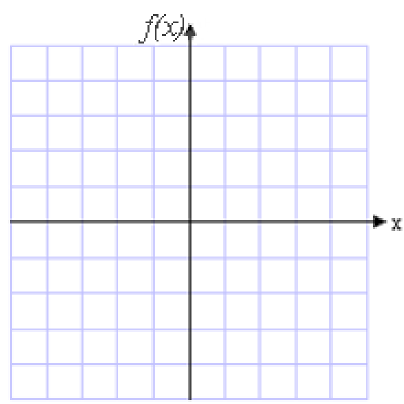
Practice:

1. Graph the following functions without the table of values.

a) $f(x) = |x+2| + 1$



b) $f(x) = [\![x]\!] - 1$



c) $f(x) = (x-1)^3 + 2$