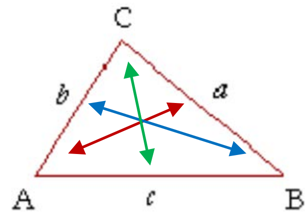
14.6 The Law of Sines and Cosines and Its Applications

oblique triangle – a triangle other then right triangle

Remember! Definitions of trig functions (**SOH-CAH-TOA**) or the **Pythagorean Theorem** can **NOT** be used when solving **oblique triangles**.

SINE LAW:

In any triangle $\triangle ABC$, the SIDES of the triangle are to one another in SINES of their opposite angles.



$$\frac{a}{\sin\angle A} = \frac{b}{\sin\angle B} = \frac{c}{\sin\angle C}$$

or equivalently

$$\frac{\sin\angle A}{a} = \frac{\sin\angle B}{b} = \frac{\sin\angle C}{c}$$

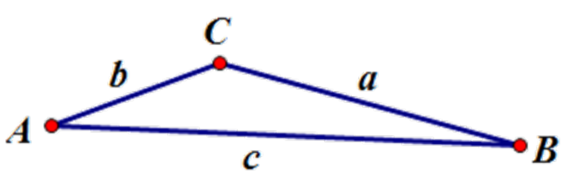
Remark! Use the Sine Law when a **pair of opposite data** is given (opposite angle and side)

Example 1:

Solve the $\triangle ABC$, if $\angle A = 20°$, $a = 11$, and $b = 7$.

Solution:

$$\frac{\sin 20°}{11} = \frac{\sin\angle B}{7}$$



$$\sin\angle B = \frac{7\sin 20°}{11} =$$

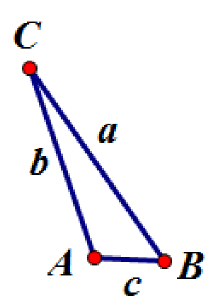
$$\angle B = \sin^{-1}(\qquad\qquad) =$$

Now $\angle C = 180° - \angle A - \angle B =$

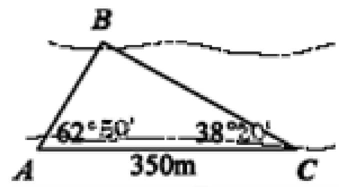
To find side c , apply the sine law again: $\frac{c}{\sin\quad°} = \frac{11}{\sin 20°}$ so $c =$ ....

Example 2:

Solve the $\triangle ABC$, if $\angle B = 65°$, $\angle C = 15°$, and $b = 12$.



Example 3:

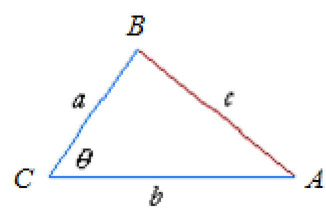


Two houses are located on opposite sides of a river at points A and B. In order to measure the distance between the houses, a measurement was taken from a third point C, located 350 m from A, on the same side of the river. Find the distance between the two houses if $\angle BAC = 62°50'$ and $\angle ACB = 38°20'$

Sometimes we need to solve a triangle with no pair of opposite data given. This happens when either

* all sides but no angles are given, or
* the given angle is enclosed by two given sides

In such cases we use the **COSINE LAW**:



$$\boldsymbol{c^2 = a^2 + b^2 - 2ab\cos\angle C}$$

Notice: The Cosine Law is an extension of The Pythagorean Theorem.

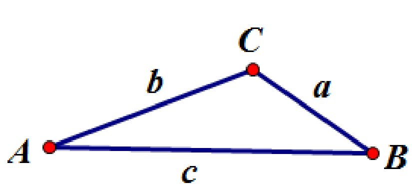
The Cosine Law can be written in different versions:

$$b^2 = a^2 + c^2 - 2ac\cos\angle B$$
$$a^2 = b^2 + c^2 - 2bc\cos\angle A$$

Example 4:

Solve the $\triangle ABC$, if $a = 5$, $b = 7$, and $c = 10$.

Solution:



Important advice:

It's easier to use the Sine Law, so use the Cosine Law only when it is absolutely necessary.

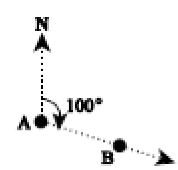
When using Cosine Law, **start with the largest angle first!**

Example 5:

Solve the $\triangle ABC$, if $a = 20$, $\angle B = 110°$ and $c = 35$.

Solution:

direction $\boldsymbol{\alpha}°$ - terminal side of the clockwise angle of $\alpha°$ that starts from the NORTH direction; for example B is in direction of 100° from A

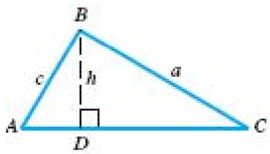


Example 6:

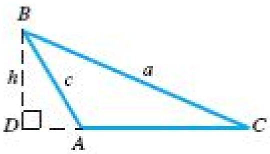
Two planes leave an airport at the same time and fly for two hours. Plane A flies in the direction of $125°$ at 425 km/h and plane B flies in the direction of $230°$ at 450 km/h. How far apart are they at this time?

Solution:

Area of a triangle:



Acute triangle ABC



Obtuse triangle ABC

Notice that $\frac{h}{c} = \sin \angle A$, so $h = c \cdot \sin \angle A$, therefore

$Area = \frac{1}{2} bh = \frac{1}{2} bc \cdot \sin \angle A$

Generally, if two sides of a triangle are known, say $\boldsymbol{a}$ and $\boldsymbol{b}$, and the angle between them, say $\angle \boldsymbol{C}$, then the area of the triangle is given by the formula:

$$\boldsymbol{A = \frac{1}{2} ab \cdot \sin \angle C}$$

Example 7:

Find the exact area of the given triangle.