12.1 Sequences and Series; Sigma Notation

an **infinite sequence** – any function with the set of **natural numbers** as the **domain**, for example: $a(n) = 2^n$ for $n \in \mathbb{N}$.
a **finite sequence** – any function with the domain of the form $\{\mathbf{1},\mathbf{2},\mathbf{3},\ldots,\boldsymbol{n}\}$, where $n \in \mathbb{N}$.
It is customary to denote the values of a sequence by $\boldsymbol{a_n}$, rather than $a(n)$, and call them **terms**.
The sequence with the **general term** $\boldsymbol{a_n = 2^n}$ can be shown by listing its terms in order 2, 4, 8, 16, 32, ... , 2^n, ...

Example 1: List the first 5 terms of the following sequences:

a) $a_n = 2 + 3n$ (**arithmetic** sequence)

b) $a_n = \frac{(-1)^n}{n}$ (**alternating harmonic** sequence)

c) $a_n = 80 \cdot \left(\frac{1}{2}\right)^n$ (**geometric** sequence)

d) $a_1 = a_2 = 1$, $a_n = a_{n-1} + a_{n-2}$ for $n \geq 3$ (**Fibonacci** sequence; defined recursively)

Example 2: Find a_8, and a_{15} for the sequence $a_n = (-1)^n \cdot \frac{n-1}{n+1}$

Example 3: Find the general term of the following sequences:

a) 1, 3, 9, 27, 81, ...

b) 7, 12, 17, 22, 27, ...

c) $-1, \frac{1}{4}, -\frac{1}{9}, \frac{1}{16}, -\frac{1}{25}, \ldots$

Example 4: A car loses 20% of its value each year. If a new car costs $24000, what is its value after 4 years? Give a formula for the value of this car after *n* years.

series – sum of **all terms** of a sequence;

example of an **infinite series**: $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+\cdots$ or a **finite series**: $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}$

partial sum S_n – sum of the first ***n*** terms of a sequence; example: $S_3 = 1+\frac{1}{2}+\frac{1}{4}$

To write a series in a compact way we use sigma notation, for example

$$\sum_{i=1}^{5} i = 1+2+3+4+5$$

or

$$\sum_{i=1}^{n} i = 1+2+3+\cdots+n$$

or

$$\sum_{i=1}^{\infty} i = 1+2+3+\cdots$$

where ***i*** is the **index of summation**.

Example 5: Find the sum of each series.

$a)\ \sum_{i=1}^{5}(i^2-1)$

$b)\ \sum_{k=2}^{6}(-1)^k(3k-2)$

Example 6: Write each series using summation notation.

a) $6-7+8-9+10$

b) $16+25+36+49+64$