

11.7 Introduction to Probability

experiment – an activity with observable outcome, for example

- flipping a coin,
- rolling a dice,
- observing the life span of certain population, etc.



sample space – the set of all possible outcomes of the experiment, for example:

sample space of rolling a dice = $\{1,2,3,4,5,6\}$

event – any subset of the sample space, for example:

obtaining an even number when rolling a dice = $\{2,4,6\}$

elementary event – an event that contains only one outcome

probability – any function $P: \{\text{events}\} \rightarrow [0, 1]$ assigning a real number between 0 and 1 (inclusive) for every event from the sample space S , such that

- $P(\emptyset) = 0$,
- $P(S) = 1$, and
- $P(A \cup B) = P(A) + P(B)$ for any **mutually exclusive** events A and B ($A \cap B = \emptyset$)

Notice: If all outcomes in a sample space S are **equally likely** (have the same probability), then the probability of an event A is

$$P(A) = \frac{|A|}{|S|} = \frac{\text{number of favorable outcomes}}{\text{number of all possible outcomes}},$$

where $|A|$ represents the number of elements in A (**cardinality** of A).

Example 1: A coin is tossed 3 times.

a) Find the sample space for this experiment.

b) Find the favorable outcomes of the event of obtaining exactly 2 heads.

c) Find the probability that we get exactly 2 heads in this experiment.

independent events – the outcome of the first event does not influence the outcome of the second event; for example: repeating the same experiment, like tossing a coin; performing two independent events, like tossing a coin and rolling a dice;

Probability Rule for Independent Events A and B :

$$P(A \cap B) = P(A) \cdot P(B)$$

Example 2: Suppose we select, without looking, two marbles from a bag containing 4 red marbles and 10 green marbles. What is the probability of selecting marbles of two different colours if

- we select them with replacement (independent events),
- we select them without replacement (dependent events).

Example 3: A committee of four is chosen from three accountants and five actuaries. Find the probability that the committee has exactly two accountants.

Binomial Probability:

If an experiment consists of n independent trials with probability p of **success** in a single trial, then the probability of k successes is given by the formula $\binom{n}{k} p^k q^{n-k}$, where $q = 1 - p$ is the probability of **failure**.

Example 4: A dice is rolled 10 times.

- Find the probability that we get 6 exactly 3 times.
- Find the probability that we get 6 at least 3 times.

Addition Rule for Probabilities:

For any events A and B , we have

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

Example 4: What is the probability of drawing an ace or a diamond from a well-shuffled deck of cards?

