Word Problems

Math 076

Word Problems

The problems in this unit can be broken down into four different types.

- Number Problems
- Geometry Problems
- Mixture Problems
- Motion Problems

Number Problems

Example 1: Three more than twice a number is forty-seven. What is the number?

Solution: With this type of problem we want to 'translate' this English sentence into an algebraic equation.

Let x be the unknown number.

"Twice a number" translates to "2x".

"Three more than…" means that we are going to be adding three to something (in this case, we are adding three to 2x)

"is" translates to "="

Putting this together we get:

Three more than twice a number is forty-seven

$$2x + 3 = 47$$

And solve: $\quad -3 \quad -3$ ⟸ Subtract 3 from both sides

$$\frac{2x}{2} = \frac{44}{2}$$ ⟸ Divide both sides by 2

$$x = 22$$

Example 2: Four less than six times a number is the same as eight less than twice that same number. Find the number.

Solution: Four less than six times a number is the same as eight less than twice that same number.

$$6x - 4 = 2x - 8$$

Now we solve: $6x - 4 = 2x - 8$

$-2x \qquad -2x$ Subtract 2x from both sides to get the x's together

$4x - 4 = -8$

$+4 \qquad +4$ Add 4 to both sides

$\frac{4x}{4} = \frac{-4}{4}$ Divide both sides by 4

$\boxed{x = -1}$

Example 3: Three times four less than twice a number is fifteen. Find the number.

Solution: Three times four less than twice a number is fifteen.

$3\,(2x - 4) = 15$

Now we solve: $3(2x - 4) = 15$ Multiply the three into the brackets

$6x - 12 = 15$

$+12 \qquad +12$ Add 12 to both sides

$6x = 27$

Example 4: Three less than seven times a number is six times the sum of that number and three. Find the number.

Solution: Three less than seven times a number is six times the sum of that number and three.

$7x - 3 = 6\,(x + 3)$

Now we solve:

$7x - 3 = 6(x + 3)$ Expand

$7x - 3 = 6x + 18$

$-6x \qquad -6x$ Subtract 6x from both sides

$x - 3 = 18$

$+3 \qquad +3$ Add 3 to both sides

$x = 21$

Example 5: The sum of three consecutive integers is 132. Find the integers.

Note: Consecutive means "in a row". For example; 7,8,9 or 10,11,12.

Therefore, if the first number is x, then the next number must x+1, and the next number must be x+2.

Solution: Call the integers x, x+1, and x+2.

Then, $x + (x + 1) + (x + 2) = 132$ Note that the brackets aren't really doing anything.

$x + x + 1 + x + 2 = 132$ Now, we can combine like terms.

$3x + 3 = 132$
-3 -3 Subtract 3 from both sides.

$\frac{3x}{3} = \frac{129}{3}$ Divide both sides by 3

$x = 43$

Therefore, the three numbers are 43,44,45.

Example 6: The sum of two consecutive odd integers is 104. Find the integers

Solution:

Call the first integer x. Then the second one must be x+2.

For example, 7 and 9 are consecutive odd integers. The second one must be 2 larger than the first one.

Then $x + x + 2 = 104$

$2x + 2 = 104$
-2 -2

$2x = 102$

$x = 51$ Therefore, the numbers must be 51 and 53.

Number Problem Exercises

1. When twice an unknown number is added to thirteen, the sum is twenty-five. What is the unknown number?
2. When twenty-five is added to three times an unknown number, the sum is thirty-four. What is the unknown number?
3. Five times an unknown number, decreased by 8 is 22. What is the unknown number?
4. Four times an unknown number, decreased by five is fifteen. What is the number?
5. An unknown number divided by twelve equals six. What is the unknown number?
6. The quotient of an unknown number and five is ten. What is the unknown number?
7. Seven minus an unknown number is equal to the unknown number plus one. What is the unknown number?

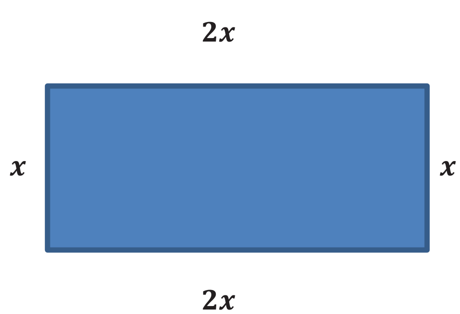
8. Six plus an unknown number is equal to twelve decreased by the unknown number. What is the unknown number?
9. When seven is added to an unknown number, the result is twice that unknown number. What is the unknown number?
10. When three times an unknown number is subtracted from twenty, the result is the unknown number. What is the number?
11. What is the unknown number if twice the sum of five and the unknown number is equal to 26?
12. What is the unknown number if four times the sum of nine and the unknown number is equal to twenty?
13. Twice the sum of four and an unknown number is equal to ten less than the unknown number. What is the unknown number?
14. Six subtracted from an unknown number is equal to three times the difference of the unknown number and eight. What is the unknown number?
15. What are the three consecutive integers whose sum is 63?
16. Find three consecutive odd integers whose sum is 21?
17. There are three consecutive even integers such that the sum of the first two integers minus the third integer is four. What are the integers?
18. There are two consecutive integers such that twice the first integer minus the second integer is eight. What are the integers?
19. There are three consecutive odd integers such that the sum of the first two integers is equal to four times the third. What are the integers?
20. There are three consecutive even integers such that the sum of the first two integers is equal to three times the third. What are the integers?

Geometry Problems

Example 1: The length of a rectangle is twice the width. The perimeter is 126 cm. Find the dimensions of the rectangle.

> Tip: With geometry problems it almost always helps to draw a picture and label the key information.

Solution: Call the width x. Then the length is 2x (because we know that the length is twice the width).



Perimeter is the total distance around the rectangle so we know that if we add up all four sides of this rectangle we should get 126 cm. This gives us an equation:

$$x + 2x + x + 2x = 126$$ ⟸ Now, combine like terms

$$6x = 126$$ ⟸ Divide both sides by 6

$$x = 21$$ ⟸ This is the width

To find the length, we know that the length is 2x. Therefore, the length is:

2x =2(21 cm)=42cm

The rectangle is 21cmx42cm.

Example 2: The area of a rectangle is 48 cm^2. The width is 9.6 cm. Find the length.

Solution: $A = l \times w$ ⟸ The formula for the area of a rectangle.

$$48 = l \times 9.6$$

$$\frac{48}{9.6} = \frac{l \times 9.6}{9.6}$$ ⟸ Divide both sides by 9.6

$$l = 5cm$$

Geometry Problem Exercises

1. The length of a rectangle is 12 inches. The perimeter is 36 inches. What is the width?
2. The perimeter of a square is 64 cm. What is the length of a side of the square?
3. The length of a rectangle is three feet more than the width. If the perimeter is 50 feet, what are the length and the width?
4. The width of a rectangle is six inches less than the length. What are the width and the length if the perimeter is 40 inches?
5. The area of a triangle is 24 m^2 and the base measures 8 m. What is the height of the triangle?
6. The length of a rectangle is three inches less than twice the width. If the perimeter is 24 inches, what are the length and the width?

Mixture Problems

These problems generally involve mixing two different substances to create a mixture. It is helpful to organize the information into a table that distinguishes between substance 1, substance 2, and the mixture.

Example 1: A retailer mixes Kenyan coffee beans with Columbian coffee beans to create 80 pounds of a coffee blend. The Kenyan beans cost the retailer $12 per pound while the Columbian coffee beans cost the retailer $9 per pound. How many pounds of each kind of bean should the retailer use so that the mixture costs him $11 per pound?

Solution: Let's start by creating a table to organize the data. We have Kenyan beans, Columbian beans, and a mixture. Give each of these their own row in the table. Then each of these has a cost per pound, a weight, and a total cost. Make these three categories into columns.

	Cost per pound	Number of pounds	Total cost
Kenyan Beans			
Columbian Beans			
Mixture			

Now fill in the information that is given in the question:

	Cost per pound	Number of pounds	Total cost
Kenyan Beans	12	x	12x
Columbian Beans	9	80-x	9(80-x)
Mixture	11	80	11(80)

Since we are looking for the number of pounds of Kenyan beans let's call this x. Note that we could easily call the number of pounds of Columbian beans x and it would also work out.

If the number of pounds of Kenyan beans is x then the number of pounds of Columbian beans must be 80-x.

To see how this works, imagine that there are 20 pounds of Kenyan beans. Then there must be 60 pounds of Columbian beans (80-20). If there are 30 pounds of Kenyan beans then there must be 50 pounds of Columbian beans (80-30). In order for them to add to 80, the Columbian beans must be 80 – x.

$$Total\ cost = (cost\ per\ pound) \times (number\ of\ pounds)$$

Now it is important to note that:

$$Total\ cost\ of\ Kenyan\ Beans + Total\ cost\ of\ Columbian\ Beans = Total\ cost\ of\ Mixture$$

So, we get the equation:

$$12x + 9(80 - x) = 11(80)$$

We can use our algebra skills to solve this.

$$12x + 9(80 - x) = 11(80)$$

$$12x + 720 - 9x = 880$$ Expand

$$3x + 720 = 880$$ Collect like terms

$$3x = 160$$ Subtract 720 from both sides

$$x = \frac{160}{3} \approx 53.33\text{pounds}$$

Therefore, there should be 53.33 pounds of Kenyan beans in the mixture.

To find weight of the Columbian beans we need to calculate $80 - x$.

$$80 - 53.33 = 26.67 \text{ pounds}$$

Therefore, there should be 26.67 pounds of Columbian beans in the mixture.

Example 2: A farmer wants to mix 10 bushels of wheat costing $10.50 per bushel with bushels of spelt consting $15 per bushel to create a mixture that is worth $13 per bushel. How many bushels of spelt should he use?

Solution: Begin by organizing the information into a table:

	Cost per bushel	Number of bushels	Total cost
Wheat	10.50	10	10.50*10
Spelt	15	x	15x
Mixture	13	10+x	13(10+x)

Call the number of bushels of spelt x because this is what we are looking for. Then the number of bushels in the mixture = the number of bushels of wheat + the number of bushels of spelt.

$total\ cost = (cost\ per\ bushel) \times (number\ of\ bushels)$

Now,

$$Total\ cost\ of\ Wheat + Total\ cost\ of\ Spelt = Total\ cost\ of\ Mixture$$

So we get the algebraic equation:

$$10.50 * 10 + 15x = 13(10 + x)$$ Expand

$$105 + 15x = 130 + 13x$$

-13x - 13x

$$105 + 2x = 130$$

-105 -105

$$\frac{2x}{2} = \frac{25}{2}$$

$$x = 12.5 \text{ bushels of spelt}$$

Example 3: John has $3.35 in nickels, dimes and quarters. He has twice as many dimes as quarters and 7 more nickels than quarters. How many of each kind of coin does he have?

Solution: Again, we can make a table to organize the information. Call the number of quarters 'x' since the other coins are being compared to the number of quarters.

	Value per coin	Number of coins	Total value
Nickels	0.05	x+7	0.05(x+7)
Dimes	0.10	2x	0.10*2x
Quarters	0.25	x	0.25x
Mixture			3.35

$$value\ in\ nickels + value\ in\ dimes + value\ in\ quarters = total\ value\ of\ mixture$$

$$0.05(x+7) + 0.10 * 2x + 0.25x = 3.35$$

$$0.05x + 0.35 + 0.20x + 0.25x = 3.35$$

$$0.50x + 0.35 = 3.35$$

$$\frac{0.50x}{0.50} = \frac{3}{0.50}$$

$$x = 6$$

Therefore, there are 6 quarters, 12 dimes, and 13 nickels.

Example 4: How many liters of a 30% alcohol solution must be mixed with 5 Liters of an 80% alcohol solution to create a 60% alcohol solution?

Solution: Start by creating a table to organize the information again. Turn the concentrations that are given in percentages into decimal form. It is important to know:

$$\boxed{concentration \times volume = total\ amount}$$

	Concentration	Volume	Total Amount of Alcohol
30% solution	0.30	x	0.30x
80% solution	0.80	5	0.80(5)
60% solution	0.60	X+5	0.60(x+5)

Note: $Amount\ of\ alcohol\ in\ 30\%\ solution + Amount\ of\ alcohol\ in\ 80\%\ solution = Amount\ of\ Alcohol\ in\ Mixture$

This gives us the equation:

$$0.3x + 0.8(5) = 0.6(x+5)$$

$$0.3x + 4 = 0.6x + 3$$

-3 -3

$$0.3x + 1 = 0.6x$$
-0.3x -0.3x

$$\frac{1}{0.3} = \frac{0.3x}{0.3}$$

3.33=x

Therefore, we should add 3.33L of the 30% solution to 5L of the 80% solution to create a 60% solution.

Example 5: How many liters of pure apple juice should be added to 3 Liters of a mixture that is 80% apple juice to create a mixture that is 90% apple juice?

Solution: Again, create a table to organize the information.

	Concentration	Volume	Total Amount of Apple juice
Pure Apple juice	1	x	x
80% apple juice	0.80	3	0.80(3)
90% apple juice	0.90	X+3	0.90(x+3)

Now, $$Amount\ of\ AJ\ in\ pure\ solution + Amount\ of\ AJ\ in\ 80\% = Amount\ of\ AJ\ in\ 90\%$$

$$x + 0.80(3) = 0.90(x + 3)$$

$$x + 2.4 = 0.9x + 2.7$$
-0.9x -0.9x

$$0.1x + 2.4 = 2.7$$
-2.4 -2.4

$$\frac{0.1x}{0.1} = \frac{0.3}{0.1}$$

$$x = 3$$

Therefore, 3L of pure apple juice should be mixed with 3L of 80% apple juice to create a mixture that is 90% apple juice.

<u>Mixture Problem Exercises:</u>

1. John wants to make a 12 pound mixture of peanuts and cashews. Peanuts cost $4 per pound and cashews cost $8 per pound. How many pounds of each kind of nut must be used if the mixture is to cost $5 per pound?
2. Randy mixes dried figs with dried apricots to make an 8 pound mixture costing $2.70 per pound. If dried figs cost $1.80 per pound and dried apricots cost $4.20 per pound, how many pounds of each are used?

3. Jason mixed 15 pounds of English toffee candy costing \$1.25 per pound with caramels costing \$1.50 per pound. How many pounds of caramels must she use to make a mixture costing \$1.35 per pound?
4. 20 pounds of kidney beans costing \$0.45 per pound is mixed with green beans costing \$0.70 per pound. How many pounds of green beans should be used to make a mixture costing \$0.60 per pound?
5. A 10-pound mixture of nuts and raisins costs \$25. If raisins cost \$1.90 per pound and nuts \$3.40 per pound, how many pounds of each are used?
6. A 50-pound mixture of Delicious and Jonathan apples costs \$14.50. If the Delicious apples cost \$0.30 per pound and the Jonathan apples cost \$0.20 per pound, how many pounds of each kind are there?
7. Mr. Wong wants to mix 30 bushels of soybeans with corn to make a 100-bushel mixture costing \$4.85 per bushel. How much can he afford to pay for each bushel of corn if soybeans cost \$8.00 per bushel?
8. Kristin wants to mix 6 pounds of Brand A coffee with Brand B coffee to make a 10 pound mixture costing \$11.50. How much can she afford to pay per pound for Brand B if Brand A costs \$1.23 per pound?
9. Bill has 13 coins in his pocket that have a total value of 95 cents. If these coins consist of nickels and dimes, how many of each kind are there?
10. Miko has 11 coins that have a total value of \$0.85. If the coins are only nickels and dimes, how many of each kind are there?
11. Jennifer has 12 coins that have a total value of \$2.20. The coins are nickels and quarters. How many of each kind are there?
12. Angelo has 18 coins consisting of nickels and quarters. If the total value of the coins is \$2.50, how many of each kind does he have?
13. Theo has \$4.00 in nickels, dimes, and quarters. If there are 4 more quarters than nickels and 3 times as many dimes as nickels, how many of each kind of coin does he have?
14. Yoko has \$5.50 in nickels, dimes, and quarters. If there are 7 more dimes than nickels and twice as many quarters as dimes, how many of each kind of coin does she have?
15. How many cubic centimeters (cc) of a 20% solution of sulfuric acid must be mixed with 100 cc of a 50% solution to make a 25% solution of sulfuric acid?
16. How many pints of a 2% solution of disinfectant must be mixed with 5 pints of a 12% solution to make a 4% solution of disinfectant?
17. If 100 gallons of 75% glycerin solution is made up by combining a 30% glycerin solution with a 90% glycerin solution, how much of each solution must be used?
18. If 1600 cc of 10% dextrose solution is made up by combining a 20% dextrose solution with a 4% dextrose solution, how much of each solution must be used?
19. How many milliliters of water must be added to 500 ml of a 40% solution of sodium bromide to reduce it to a 25% solution? (Hint: water has 0% sodium bromide).
20. How many liters of pure alcohol must be added to 10 liters of a 20% solution of alcohol to make a 50% solution?

21. How many ounces of a 10% salt solution should be mixed with 20 ounces of a 25% solution to make a 16% solution?
22. How many milliliters of a 4% acid solution should be mixed with 100 ml of a 20% solution to make an 8% solution?
23. A chemist needs 50 ml of a 30% iodine solution. If she has only a 50% solution and a 25% solution in stock, how much of each will she need?
24. How many quarts of pure antifreeze must be added to a 20% antifreeze solution to make 8 quarts of a 50% solution?

Distance Problems

The important formula for solving distance problems is:

$$rate \times time = distance$$

Example 1: Jerry takes 2 hours to drive to work in the morning but it takes him 2.5 hours to return home by the same route. His average speed in the morning is 10 km/h faster than his average speed in the evening. How far is his work from his home?

Solution: Begin by drawing a table. There are two cases to consider. The first is on the way to work and the second is on the way home. Use rate, time, and distance from the above formula to create columns.

	Rate	Time	Distance
To work	x+10	2	2(x+10)
From work	x	2.5	2.5x

$$rate \times time = distance$$

If we call the speed from work x, then we know that his speed to work must be x+10 since he went 10 km/h faster on the way to work.

Once the table is created then we need to consider how the two distances are related. In this case, the distance to work is the same as the distance from work so we know that they are equal. Therefore, we get the equation:

$$2(x + 10) = 2.5x$$

$$2x + 20 = 2.5x$$

-2x -2x

$$\frac{20}{0.5} = \frac{0.5x}{0.5}$$

$$x = 40$$

Therefore, on his way from work he averages 40 km/h and on his way to work he averages 50 km/h (plug it into x+10 in the first row of the table).

Example 2: Two trains leave cities that are 1500 km apart and head towards each other. Train A is travelling at 100 km/h and train B is travelling at 120 km/h. How long will it take for the two trains to meet? How long will each of them have travelled?

Solution: Again, create a table to organize the information. We are looking for the time, so call it x. In this case, it is the same for both of them since they have been travelling for the same amount of time when they meet.

	Rate	Time	Distance
Train A	100	x	100x
Train B	120	x	120x

$rate \times time = distance$

Now, we need to think about how the two distances are related. In this example, the two trains travel a total distance of 1500 km. Therefore, we get the equation:

$$100x + 120x = 1500$$

$$\frac{220x}{220} = \frac{1500}{220}$$

$$x = 6.82\ hrs$$

Therefore, they have both been travelling for 6.82 hours when they meet. To find the distance that each of them travels, plug this into the distance equation for each train.

Train A: $d = 100x = 100(6.82) = 682\ km$

Train B: $d = 120x = 120(6.82) = 818\ km$

Example 3: It takes Sarah 3 hours to row her kayak downstream to a camping spot. It takes her 5 hours to get back the next day because she has to fight the current the whole way. If the river is flowing at 4 km/h, how fast could Sarah row in still water?

Solution: Call the speed of the kayak x. Then on the way downstream, the current is helping Sarah and she travels at a speed of x+4. On the way upstream, the current is fighting her and she travels at a speed of x-4. Using this we can fill in the table.

	Rate	Time	Distance
Downstream	x+4	3	3(x+4)
Upstream	x-4	5	5(x-4)

In this case, the distance to get to the camping site and the distance to get back are equal. This gives the following equation:

$$3(x + 4) = 5(x - 4)$$

$$3x + 12 = 5x - 20$$

-5x -5x

$$-2x + 12 = -20$$

-12 -12

$$\frac{-2x}{-2} = \frac{-32}{-2}$$

$$x = 16$$

Therefore, Sarah can row at a speed of 16 km/h in still water.

Distance Problem Exercises:

1. The Malone family left San Diego by car at 7 A.M. bound for San Francisco. Their neighbors, the King family, left in their car at 8 A.M. also bound for San Francisco. By travelling 9 mph faster, the Kings overtook the Malones at 1 P.M.
    a. What was the average rate of speed of each car?
    b. What was the total distance travelled by each car before they met?
2. The Duran family left Ames, Iowa, by car at 6 A.M., bound for Yellowstone National Park. Their neighbors, the Silva family, left in their car at 8 A.M. also bound for Yellowstone. By traveling 10 mph faster, the Silvas overtook the Durans at 4 P.M.
    a. What was the average speed of each car?
    b. What was the total distance traveled before they met?
3. Tran hiked from his camp to a lake in the mountains and returned to camp later that day. He walked at a rate of 2 mph going to the lake and 5 mph coming back. If the trip to the lake took 3 hours longer than the trip back:
    a. How long did it take Tran to hike to the lake?
    b. How far is it from his camp to the lake?
4. Lee hiked from her camp up to an observation tower in the mountains and returned to camp later in the day. She walked up at the rate of 2 mph and jogged back at the rate of 6 mph. The trip to the tower to 2 hours longer than the return trip.
    a. How long did it take her to hike to the tower?
    b. How far is it from her camp to the tower?
5. Anita and Felix live 54 miles apart. Both leave their homes at 7 A.M. by bicycle, riding toward each other. They meet at 10AM. If Felix's average speed is 2 mph faster than Anita's speed, how fast does each cycle?
6. Danny and Yolanda live 60 miles apart. Both leave their homes at 10 AM by bicycle, riding toward each other. They meet at 2 PM. If Yolanda's average riding speed is 3 mph slower than Danny's speed, how fast does each cycle?

7. Abdul paddled a kayak downstream for 3 hours. After having lunch, he took 5 hours to paddle back upstream. If the speed of the stream is 2 mph, how fast does Abdul row in still water? How far downstream did he travel?
8. The Chang family sailed their houseboat upstream for 4 hours, but it took them only 2 hours to sail back downstream. If the speed of the houseboat in still water is 15 mph, what was the speed of the stream? How far upstream did the Changs travel?
9. Gerrit flew his private plane from his office to his company's storage facility bucking a 20 mph head wind all the way. He flew home the same day with the same wind at his back. The round trip took 10 hours of flying time. If the plane makes 100 mph in still air, how far is the storage facility from his office?
10. A motor boat cruised from the marina to an island and then back to the marina. The speed of the boat in still water was 30 mph and the speed of the current was 6 mph. If the total trip took 5 hours, how far is it from the marina to the island?

Solutions to Exercises:

Number Problem Solutions:

1) 6 2) 3 3) 6 4) 5 5) 72 6) 50 7) 3 8) 3 9) 7 10) 5 11) 8 12) -4 13) -18 14) 9
15) 20,21,22 16) 5,7,9 17) 6,8,10 18) 9 19) -7,-5,-3 20) -10,-8,-6

Geometry Problem Solutions:

1) 6 2) 16 3) width is 11 ft, length is 14 ft 4) length is 13, width is 7 5) 6 m 6) length is 7, width is 5

Mixture Problem Solutions:

1) 9 lb peanuts, 3 lb cashews 2) 5 lb figs, 3 lb apricots 3) 10 lb 4) 30 lb
5) 4 lb of nuts, 6 lb of raisins 6) 45 lb Delicious apples, 5 lb Jonathan apples 7) $3.50 8) $1.03
9)7 nickels, 6 dimes 10) 5 nickels, 6 dimes 11) 4 nickels, 8 quarters 12) 10 nickels, 8 quarters
13) 5 nickels, 9 quarters, and 15 dimes 14) 2 nickels, 9 dimes, 18 quarters 15) 500 cc 16) 20 pints
17) 25 gal 30% solution, 75 gal 90% solution 18) 600cc of 20%, 1000 cc of 4% 19) 300 ml 20) 6 L
21) 30 oz 22) 300 mL 23) 10 ml of 50% solution, 40 ml of 25% solution 24) 3 L

Distance Problem Solutions:

1a) Malone's 45 mph, King's 54 mph 1b) 270 mi 2a) Durans 40 mph, Silvas 50 mph b) 400 miles

3a) 5 hours 3b) 10 mi 4a) 3 hours 4b) 6 miles 5) Anita's speed 8 mph, Felix's speed 10 mph

6) Danny 9 mph, Yolanda 6 mph 7) speed still water 8 mph, distance downstream = 30 mi

8a) 5 mph b) 40 miles 9) 480 miles 10) 72 miles