L.3 Applications of Linear Equations

In this section, we study some strategies of solving problems with the use of linear equations, or well-known formulas. While there are many approaches to problem solving, the following steps prove to be helpful.

Five Steps for Problem Solving
1. **Familiarize** yourself with the problem.
2. **Translate** the problem to a symbolic representation (usually an **equation** or an **inequality**).
3. **Solve** the equation(s) or the inequality(s).
4. **Check** if the answer make sense in the original problem.
5. **State the answer** to the original problem clearly.

Here are some hints of how to **familiarize** yourself with the problem:

- **Read** the problem carefully a few times. In the first reading focus on the general setting of the problem. See if you can identify this problem as one of a motion, investment, geometry, age, mixture or solution, work, or a number problem, and draw from your experiences with these types of problems. During the second reading, focus on the specific information given in the problem, skipping unnecessary words, if possible.
- **List the information** given, including **units**, and check **what the problem asks for**.
- If applicable, **make a diagram** and label it with the given information.
- **Introduce a variable**(s) for the unknown quantity(ies). Make sure that the variable(s) is/are clearly defined (including units) by writing a "let" statement or labeling appropriate part(s) of the diagram. Choose descriptive letters for the variable(s). For example, let l be the length in centimeters, let t be the time in hours, etc.
- Express **other unknown values** in terms of the already introduced variable(s).
- Write applicable **formulas**.
- **Organize your data** in a meaningful way, for example by filling in a table associated with the applicable formula, inserting the data into an appropriate diagram, or listing them with respect to an observed pattern or rule.
- **Guess** a possible answer and check your guess. Observe the way in which the guess is checked. This may help you translate the problem to an equation.

Translation of English Phrases or Sentences to Expressions or Equations

One of the important phases of problem solving is **translating** English words into **symbolic representation**.

Here are the most commonly used **key words** suggesting a particular operation:

ADDITION (+)	SUBTRACTION (−)	MULTIPLICATION (·)	DIVISION (÷)
sum	**difference**	**product**	**quotient**
plus	minus	multiply	divide
add	**subtract from**	times	ratio
total	**less than**	**of**	**out of**
more than	less	half of	per
increase by	decrease by	half as much as	shared
together	diminished	twice, triple	cut into
perimeter	shorter	area	

Example 1 **Translating English Words to an Algebraic Expression or Equation**

Translate the word description into an algebraic expression or equation.

a. The sum of half of a number and two
b. The square of a difference of two numbers
c. Triple a number, increased by five, is one less than twice the number.
d. The quotient of a number and seven diminished by the number
e. The quotient of a number and seven, diminished by the number
f. The perimeter of a rectangle is four less than its area.
g. In a package of 12 eggs, the ratio of white to brown eggs is one out of three.
h. Five percent of the area of a triangle whose base is one unit shorter than the height

Solution

a. Let x represents "*a number*". Then

*The **sum of** half of a number **and** two* translates to $\frac{1}{2}x + 2$

Notice that the word "***sum***" indicates addition sign at the position of the word "***and***". Since addition is a binary operation (needs two inputs), we reserve space for "*half of a number*" on one side and "*two*" on the other side of the addition sign.

b. Suppose x and y are the "*two numbers*". Then

***The square of** a **difference of** two numbers* translates to $(x-y)^2$

Notice that we are squaring everything that comes after "*the square of*".

c. Let x represents "*a number*". Then

***Triple** a number, increased by five, **is** one **less than** **twice** the number.*

translates to the equation: $3x + 5 = 2x - 1$

This time, we translated a sentence that results in an equation rather than expression. Notice that the "equal" sign is used in place of the word "is". Also, remember that phrases "less than" or "subtracted from" work "backwards". For example A **less than** B or A **subtracted from** B translates to $B - A$. However, the word "less" is used in the usual direction, from left to right. For example, A **less** B translates to $A - B$.

d. Let x represent "*a number*". Then

***The quotient of** a number **and** seven **diminished** by the number* translates to $\frac{x}{7-x}$

Notice that "**the** number" refers to the same number x.

e. Let x represent "*a number*". Then

***The quotient of** a number **and** seven**,** **diminished** by the number* translates to $\frac{x}{7} - x$

Here, the **comma** indicates the end of the "**quotient section**". So, we diminish the quotient rather than diminishing the seven, as in *Example 1d*.

f. Let l and w represent the length and the width of a rectangle. Then

*The **perimeter** of a rectangle **is** four **less than** its **area**.*

translates to the equation: $\boldsymbol{2l+2w=lw}-4$

Here, we use a formula for the perimeter ($\boldsymbol{2l+2w}$) and for the area ($\boldsymbol{lw}$) of a rectangle.

g. Let w represent the number of white eggs in a package of 12 eggs. Then $(12-w)$ represents the number of brown eggs in this package. Therefore,

*In a package of 12 eggs, **the ratio of the number of white eggs to the number of brown eggs is the same as two to three**.*

translates to the equation: $\boldsymbol{\frac{w}{12-w}=\frac{2}{3}}$

Here, we expressed the unknown number of brown eggs $(12-w)$ in terms of the number w of white eggs. Also, notice that the order of listing terms in a proportion is essential. Here, the first terms of the two ratios are written in the numerators (in blue) and the second terms (in brown) are written in the denominators.

h. Let h represent the height of a triangle. Since the base is *one unit shorter than the height*, we express it as $(h-1)$. Using the formula $\frac{1}{2}bh$ for area of a triangle, we translate

five percent of the area of a triangle** whose **base is one unit shorter than the height

to the expression: $\boldsymbol{0.05\cdot\frac{1}{2}(h-1)h}$

Here, we convert *five percent* to the number 0.05, as *per-cent* means *per hundred*, which tells us to divide 5 by a hundred.
Also, observe that the above word description is not a sentence, even though it contains the word "*is*". Therefore, the resulting symbolic form is an expression, not an equation. The word "*is*" relates the base and the height, which in turn allows us to substitute $(h-1)$ in place of b, and obtain an expression in one variable.

So far, we provided some hints of how to familiarize ourself with a problem, we worked through some examples of how to translate word descriptions to a symbolic form, and we reviewed the process of solving linear equations (*see section L1*). In the rest of this section, we will show various methods of solving commonly occurring types of problems, using representative examples.

Number Relation Problems

In number relation type of problems, we look for relations between quantities. Typically, we introduce a variable for one quantity and express the other quantities in terms of this variable following the relations given in the problem.

Example 2 ► **Solving a Number Relation Problem with Three Numbers**

The sum of three numbers is forty-two. The second number is twice the first number, and the third number is three less than the second number. Find the three numbers.

Solution ► There are three unknown numbers such that their sum is forty-two. This information allows us to write the equation

$$1^{\text{st}} \text{ number} + 2^{\text{nd}} \text{ number} + 3^{\text{rd}} \text{ number} = 42.$$

To solve such an equation, we wish to express all three unknown numbers in terms of one variable. Since the second number refers to the first, and the third number refers to the second, which in turn refers to the first, it is convenient to introduce a variable for the first number.
So, let n represent the first number.
The second number is twice the first, so $2n$ represents the second number.
The third number is three less than the second number, so $2n - 3$ represents the third number.

Therefore, our equation turns out to be

$$\begin{aligned} n + 2n + (2n - 3) &= 42 \\ 5n - 3 &= 42 \quad && / +3 \\ 5n &= 45 \quad && / \div 5 \\ n &= 9. \end{aligned}$$

Hence, the first number is **9**, the second number is $2n = 2 \cdot 9 = \mathbf{18}$, and the third number is $2n - 3 = 18 - 3 = \mathbf{15}$.

Consecutive Numbers Problems

Since **consecutive numbers** differ by one, we can represent them as $\boldsymbol{n, n+1, n+2}$, and so on.

Consecutive even or **consecutive odd** numbers differ by two, so both types of numbers can be represented by $\boldsymbol{n, n+2, n+4}$, and so on.

Notice that if the first number n is even, then $n + 2, n + 4, \ldots$ are also even; however, if the first number n is odd then $n + 2, n + 4, \ldots$ are also odd.

Example 3 ► **Solving a Consecutive Odd Integers Problem**

Find three consecutive odd integers such that three times the middle integer is seven more than the sum of the first and third integers.

Solution ► Let the three consecutive odd numbers be called $n, n + 2$, and $n + 4$. We translate *three times the middle integer is seven more than the sum of the first and third integers* into the equation

$$3(n + 2) = n + (n + 4) + 7$$

which gives

$$3n + 6 = 2n + 11 \qquad / \ -6, -2n$$

$$n = 5$$

Hence, the first number is **5**, the second number is $n + 2 = \mathbf{7}$, and the third number is $n + 4 = \mathbf{9}$.

Percent Problems

Rules to remember when solving percent problems:

$$1 = 100\% \quad \text{and} \quad \frac{\textit{is a part}}{\textit{of a whole}} = \frac{\%}{100}$$

Also, remember that

$$\textit{percent increase}(\textit{decrease}) = \frac{\textit{last} - \textit{first}}{\textit{first}} \cdot 100\%$$

Example 4 ► **Finding the Amount of Tax**

Joe bought a new computer for \$1506.75, including sales tax at 5%. What amount of tax did he pay?

Solution ► Suppose the computer cost p dollars. Then the tax paid for this computer is 5% of p dollars, which can be represented by the expression $0.05p$. Since the total cost of the computer with tax is \$1506.75, we set up the equation

$$p + 0.05p = 1506.75$$

which gives us

$$1.05p = 1506.75 \qquad / \ \div 1.05$$

$$p = 1435$$

The question calls for the amount of tax, so we calculate $0.05p = 0.05 \cdot 1435 = 71.75$.

Joe paid \$71.75 of tax for his computer.

Example 5 ► **Solving a Percent Increase Problem**

After 1 yr on the job, Garry got a raise from \$10.50 per hour to \$11.34 per hour. What was the percent increase in his hourly wage?

Solution ► We calculate the percent increase by following the rule $\frac{\textit{last}-\textit{first}}{\textit{first}} \cdot 100\%$.

So, Garry's hourly wage was increased by $\frac{11.34-10.50}{10.50} \cdot 100\% = \mathbf{8}\%$.

Investment Problems

When working with investment problems we often use the simple interest formula $\boldsymbol{I = Prt}$, where I represents the amount of interest, P represents the principal (amount of money invested), r represents the interest rate, and t stands for time in years.

Also, it is helpful to organize data in a diagram like this:

total amount

account ***I*** account ***II***

$$\textbf{interest } from\ account\ \boldsymbol{I} + \textbf{interest } from\ account\ \boldsymbol{II} = \textbf{total interest}$$

or

$$\textbf{interest } from\ account\ \boldsymbol{I} = \textbf{interest } from\ account\ \boldsymbol{II}$$

Example 6 ► **Solving an Investment Problem**

A student takes out two student loans, one at 6% annual interest and the other at 4% annual interest. The total amount of the two loans is \$5000, and the total interest after 1 year is \$244. Find the amount of each loan.

Solution ► To solve this problem in one equation, we would like to introduce only one variable. Suppose $\boldsymbol{x}$ is the amount of the first student loan. Then the amount of the second student loan is the remaining portion of the \$5000. So, it is $(5000 - \boldsymbol{x})$.

Using the simple interest formula $I = Prt$, for $t = 1$, we calculate the interest obtained from the 6% to be $0.06 \cdot \boldsymbol{x}$ and from the 4% account to be $0.04(5000 - \boldsymbol{x})$. The equation arises from the fact that the total interest equals to \$244, as indicated in the diagram below.

\$5000

$\boldsymbol{x}$ at 6% $(5000 - \boldsymbol{x})$ at 4%

$$0.06 \cdot \boldsymbol{x} + 0.04(5000 - \boldsymbol{x}) = 244$$

For easier calculations, we may want to clear decimals by multiplying this equation by 100. This gives us

$$6x + 4(5000 - x) = 24400$$

$$6x + 20000 - 4x = 24400 \quad / -20000$$

$$2x = 4400 \quad / \div 2$$

and finally

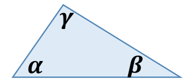
$$x = \$2200$$

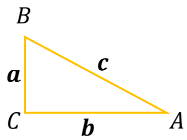
Thus, the first loan is **\$2200** and the second loan is $5000 - x = 5000 - 2200 = \$\mathbf{2800}$.

Geometry Problems

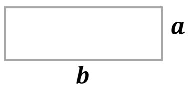
In geometry problems we often use well-known formulas or facts that pertain to geometric figures. Here is a list of facts and formulas that are handy to know when solving various problems.

- The **sum of angles** in a triangle equals **180°**.



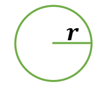


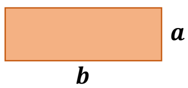
- The lengths of sides in a right-angle triangle ABC satisfy the **Pythagorean equation** $\boldsymbol{a^2+b^2=c^2}$, where c is the hypotenuse of the triangle.



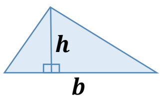
- The **perimeter of a rectangle** with sides a and b is given by the formula $\boldsymbol{2a+2b}$.

- The **circumference** of a circle with radius r is given by the formula $\boldsymbol{2\pi r}$.

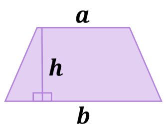




- The **area of** a **rectangle** or a **parallelogram** with base b and height h is given by the formula $\boldsymbol{bh}$.

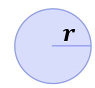


- The **area of** a **triangle** with base b and height h is given by the formula $\boldsymbol{\frac{1}{2}bh}$.



- The **area of** a **trapezoid** with bases a and b, and height h is given by the formula $\boldsymbol{\frac{1}{2}(a+b)h}$.

- The **area of** a **circle** with radius r is given by the formula πr^2.

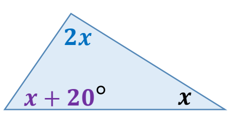


Example 7 ► **Finding the Measure of Angles in a Triangle**

In a triangular cross section of a roof, the second angle is twice as large as the first. The third angle is 20° greater than the first angle. Find the measures of the angle

Solution ►

Observe that the size of the second and the third angle is compared to the size of the first angle. Therefore, it is convenient to introduce a variable, $\boldsymbol{x}$, for the measure of the first angle. Then, the expression for the measure of the second angle, which *is twice as large as the first*, is $\boldsymbol{2x}$ and the expression for the measure of the third angle, which is *20° greater than the first*, is $\boldsymbol{x+20°}$. To visualize the situation, it might be helpful to draw a triangle and label the three angles.



Since the sum of angles in any triangle is equal to 180°, we set up the equation

$$x+2x+x+20°=180°$$

This gives us

$$4x+20°=180° \qquad /\ -20°$$

$$4x=160° \qquad /\ \div 4$$

$$x=40°$$

So, the measure of the first angle is $\boldsymbol{40°}$,
the measure of the second angle is $2x=2\cdot 40°=\boldsymbol{80°}$, and
the measure of the third angle is $x+20°=40°+20°=\boldsymbol{60°}$.

Total Value Problems

When solving total value types of problems, it is helpful to organize the data in a table that compares the number of items and the value of these items. For example:

	item ***A***	item ***B***	total
number of items			
value of items			

Example 8 ► **Solving a Coin Problem**

A collection of twenty-two coins has a value of \$4.75. The collection contains dimes and quarters. Find the number of quarters in the collection.

Solution ► Suppose the number of quarters is n. Since the whole collection contains 22 coins, then the number of dimes can be represented by $22 - n$. Also, in cents, the value of n quarters is $25n$, while the value of $22 - n$ dimes is $10(22 - n)$. We can organize this information as in the table below.

	dimes	quarters	Total
number of coins	$22 - n$	n	22
value of coins (in cents)	$10(22 - n)$	$25n$	475

The value is written in cents!

Using the last row of this table, we set up the equation

$$10(22 - n) + 25n = 475$$

and then solve it for n.

$$220 - 10n + 25n = 475 \qquad / -220$$
$$15n = 255 \qquad / \div 15$$
$$n = 17$$

So, there are **17** quarters in the collection of coins.

Mixture-Solution Problems

When solving total mixture or solution problems, it is helpful to organize the data in a table that follows one of the formulas

$unit\ price \cdot number\ of\ units = total\ value$ or $percent \cdot volume = content$

	unit price ·	# of units	= value
type I			
type II			
mix			

	% ·	volume	= content
type I			
type II			
solution			

Example 9 ► **Solving a Mixture Problem**

A mixture of nuts was made from peanuts that cost \$3.60 per pound and cashews that cost \$9.00 per pound. How many of each type of nut were used to obtain a 60 pounds of mixture that costs \$5.40 per pound?

Solution ► In this problem, we mix two types of nuts: peanuts and cashews. Let x represent the number of pounds of peanuts. Since there are 60 pounds of the mixture, we will express the number of pounds of cashews as $60 - x$.

The information given in the problem can be organized as in the following table.

	unit price ·	# of units	= value (in \$)
peanuts	3.60	x	$3.6x$
cashews	9.00	$60 - x$	$9(60 - x)$
mix	5.40	60	324

To complete the last column, multiply the first two columns.

Using the last column of this table, we set up the equation

$$3.6x + 9(60 - x) = 324$$

and then solve it for x.

$$3.6x + 540 - 9x = 360 \qquad / -540$$
$$-5.4x = -216 \qquad / \div(-5.4)$$
$$x = 40$$

So, there were **40** pounds of peanuts and $60 - x = 60 - 40 = \mathbf{20}$ pounds of cashews used for the mix.

Example 10 ► **Solving a Solution Problem**

How many milliliters of pure acid must be added to 60 ml of an 8% acid solution to make a 20% acid solution?

Solution ► Let x represent the volume of the pure acid, in milliliters. The 20% solution is made by combining x ml of the pure acid with 60 ml of an 8% acid, so the volume of the solution can be expressed as $x + 60$.

Now, let us organize this information in the table below.

	% ·	volume	= acid
pure acid	1	x	x
8% acid	0.08	60	4.8
solution	0.20	$x + 60$	$0.2(x + 60)$

To complete the last column, multiply the first two columns.

Using the last column of this table, we set up the equation

$$x + 4.8 = 0.2(x + 60)$$

and then solve it for x.

$$x + 4.8 = 0.2x + 12 \qquad / -0.2x, -4.8$$
$$0.8x = 7.2 \qquad / \div 0.8$$
$$x = 9$$

So, there must be added **9** milliliters of pure acid.

Motion Problems

When solving motion problems, refer to the formula

$$\boldsymbol{Rate \cdot Time = Distance}$$

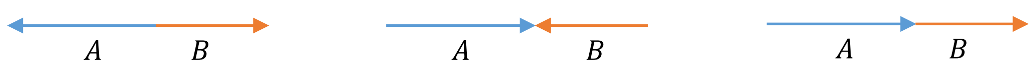
and organize date in a table like this:

	R ·	T	= D
motion I			
motion II			
total			

Some boxes in the "total" row are often left empty. For example, in motion problems, we usually do not add rates. Sometimes, the "total" row may not be used at all.

If two moving object (or two components of a motion) are analyzed, we usually encounter the following situations:

1. The two objects A and B move apart, approach each other, or move successively in the same direction (see the diagram below). In these cases, it is likely we are interested in the **total distance** covered. So, the last row in the above table will be useful to record the total values.

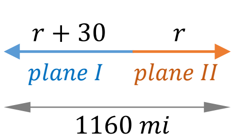


2. Both objects follow the same pathway. Then the **distances** corresponding to the two motions are the same and we may want to **equal** them. I such cases, there may not be any total values to consider, so the last row in the above table may not be used at all.

Example 11 ► **Solving a Motion Problem where Distances Add**

Two small planes start from the same point and fly in opposite directions. The first plane is flying 30 mph faster than the second plane. In 4 hours, the planes are 2320 miles apart. Find the rate of each plane.

Solution ► The rates of both planes are unknown. However, since the rate of the first plane is 30 mph faster than the rate of the second plane, we can introduce only one variable. For example, suppose r represents the rate of the second plane. Then the rate of the first plane is represented by the expression $r + 30$.



In addition, notice that 1160 miles is the **total distance** covered by both planes, and 4 hours is the flight time of each plane.

Now, we can complete a table following the formula $R \cdot T = D$.

	R ·	T	= D
plane I	$r + 30$	4	$4(r + 30)$
plane II	r	4	$4r$
total			1160

Notice that neither the total rate, nor the total time was included here. This is because these values are not relevant for this particular problem. The equation that relates distances comes from the last column:

$$4(r + 30) + 4r = 1160$$

After solving it for r,

$$4r + 120 + 4r = 1160 \qquad / -120$$

$$8r = 1040 \qquad / \div 8$$

we obtain

$$r = 130$$

Therefore, the speed of the first plane is $r + 30 = 130 + 30 =$ **160 mph** and the speed of the second plane is **130 mph**.

Example 12 ► **Solving a Motion Problem where Distances are the Same**

A speeding car traveling at 80 mph passes a police officer. Ten seconds later, the police officer chases the speeding car at a speed of 100 mph. How long, in minutes, does it take the police officer to catch up with the car?

Solution ► Let t represent the time, in minutes, needed for the police officer to catch up with the car. The time that the speeding car drives is 10 seconds longer than the time that the police officer drives. To match the denominations, we convert 10 seconds to $\frac{10}{60} = \frac{1}{6}$ of a minute. So, the time used by the car is $t + \frac{1}{6}$.

In addition, the rates are given in miles per hour, but we need to express them as miles per minute. We can convert $\frac{80\ mi}{1\ h}$ to, for example, $\frac{80\ mi}{60\ min} = \frac{4}{3}\frac{mi}{min}$, and similarly, $\frac{100\ mi}{1\ h}$ to $\frac{100\ mi}{60\ min} = \frac{5}{3}\frac{mi}{min}$.
Now, we can complete a table that follows the formula $R \cdot T = D$.

	R ·	T	= D
car	$\frac{4}{3}$	$t + \frac{1}{6}$	$\frac{4}{3}(t + \frac{1}{6})$
police	$\frac{5}{3}$	t	$\frac{5}{3}t$

Notice that this time there is no need for the "total" row.

Since distances covered by the car and the police officer are the same, we set up equation

$$\frac{4}{3}\left(t + \frac{1}{6}\right) = \frac{5}{3}t \qquad / \cdot 3$$

To solve it for t, we may want to clear some fractions first. After multiplying by 3, we obtain

$$4\left(t+\frac{1}{6}\right)=5t$$

which becomes

$$4t+\frac{2}{3}=5t \qquad / -4t$$

and finally

$$\frac{2}{3}=t$$

So, the police officer needs $\frac{2}{3}$ of a minute (which is 40 seconds) to catch up with the car.

Even though the above examples show a lot of ideas and methods used in solving specific types of problems, we should keep in mind that the best way to learn problem solving is to **solve a lot of problems**. This is because every problem might present slightly different challenges than the ones that we have seen before. The more problems we solve, the more experience we gain, and with time, problem solving becomes easier.

L.3 Exercises

Concept Check *Translate each word description into an algebraic expression or equation.*

1. A number less seven

2. A number less than seven

3. Half of the sum of two numbers

4. Two out of all apples in the bag

5. The difference of squares of two numbers

6. The product of two consecutive numbers

7. The sum of three consecutive integers is 30.

8. Five more than a number is double the number.

9. The quotient of three times a number and 10

10. Three percent of a number decreased by a hundred

11. Three percent of a number, decreased by a hundred

12. The product of 8 more than a number and 5 less than the number

13. A number subtracted from the square of the number

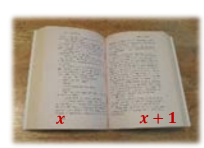
14. The product of six and a number increased by twelve

Concept Check *Solve each problem.*

15. If the quotient of a number and 6 is added to twice the number, the result is 8 less than the number. Find the number.

16. When 75% of a number is added to 6, the result is 3 more than the number. Find the number.

17. The sum of the numbers on two adjacent post-office boxes is 697. What are the numbers?



18. The sum of the page numbers on a pair of facing pages of a book is 543. What are the page numbers?

19. The enrollment at a community college declined from 12,750 during one school year to 11,350 the following year. Find the percent decrease to the nearest percent.

20. Between 2010 and 2015, the population of Alaska grew from 710,231 to 737,625. What was the percent increase to the nearest tenth of a percent?

Analytic Skills *Solve each problem.*

21. Find three consecutive odd integers such that the sum of the first, two times the second, and three times the third is 70.

22. Find three consecutive even integers such that the sum of the first, five times the second, and four times the third is 1226.

23. Twice the sum of three consecutive odd integers is 150. Find the three integers.

24. Jeff knows that his neighbour Sarah paid $40,230, including sales tax, for a new Buick Park Avenue. If the sales tax rate is 8%, then what is the cost of the car before tax?

25. After a salesman receives a 5% raise, his new salary is $40,530. What was his old salary?

26. Clayton bought a ticket to a rock concert at a discount. The regular price of the ticket was $70.00, but he only paid $59.50. What was the percent discount?

27. About 182,900 patents were issued by the U.S. government in 2007. This was a decrease of about 7% from the number of patents issued in 2006. To the nearest hundred, how many patents were issued in 2006?

28. A person invests some money at 5% and some money at 6% simple interest. The total amount of money invested is $2400 and the total interest after 1 year is $130.50. Find the amount invested at each rate.

29. Thomas Flanagan has $40,000 to invest. He will put part of the money in an account paying 4% simple interest and the remainder into stocks paying 6% simple interest. The total annual income from these investments should be $2040. How much should he invest at each rate?

30. Jennifer Siegel invested some money at 4.5% simple interest and $1000 less than twice this amount at 3%. Her total annual income from the interest was $1020. How much was invested at each rate?

31. Piotr Galkowski invested some money at 3.5% simple interest, and $5000 more than three times this amount at 4%. He earned $1440 in annual interest. How much did he invest at each rate?

32. Dan Abbey has invested $12,000 in bonds paying 6%. How much additional money should he invest in a certificate of deposit paying 3% simple interest so that the total return on the two investments will be 4%?

33. Mona Galland received a year-end bonus of $17,000 from her company and invested the money in an account paying 6.5%. How much additional money should she deposit in an account paying 5% so that the return on the two investments will be 6%?

34. A piece of wire that is 100 cm long is to be cut into two pieces, each to be bent to make a square. The length of a side of one square is to be twice the length of a side of the other. How should the wire be cut?

35. The measure of the largest angle in a triangle is twice the measure of the smallest angle. The third angle is 10° less than the largest angle. Find the measure of each angle.

36. A carpenter used 30 ft of molding in three pieces to trim a garage door. If the long piece was 2 ft longer than twice the length of each shorter piece, then how long was each piece?

37. Clint is constructing two adjacent rectangular dog pens. Each pen will be three times as long as it is wide, and the pens will share a common long side. If Clint has 65 ft of fencing, what are the dimensions of each pen?

38. The width of a standard tennis court used for playing doubles is 42 feet less than the length. The perimeter of the court is 228 feet. Find the dimensions of the court.

39. Dana inserted eight coins, consisting of dimes and nickels, into a vending machine to purchase a *Snickers* bar for 55 cents. How many coins of each type did she use?

40. Ravi took eight coins from his pocket, which contained only dimes, nickels, and quarters, and bought the Sunday Edition of *The Daily Star* for 75 cents. If the number of nickels he used was one more than the number of dimes, then how many of each type of coin did he use?

41. Pecans that cost $28.50 per kilogram were mixed with almonds that cost $22.25 per kilogram. How many kilograms of each were used to make a 25-kilogram mixture costing $24.25 per kilogram?

42. A manager bought 12 pounds of peanuts for $30. He wants to mix $5 per pound cashews with the peanuts to get a batch of mixed nuts that is worth $4 per pound. How many pounds of cashews are needed?

43. Adult tickets for a play cost $10.00, and children's tickets cost $4.00. For one performance, 460 tickets were sold. Receipts for the performance were $3760. Find the number of adult tickets sold.

44. Tickets for a school play sold for $7.50 for each adult and $3.00 for each child. The total receipts for 113 tickets sold were $663. Find the number of adult tickets sold.

45. Find the cost per ounce of a sunscreen made from 100 ounces of lotion that cost $3.46 per ounce and 60 ounces of lotion that cost $12.50 per ounce.

46. A tea mixture was made from 40 lb of tea costing $5.40 per pound and 60 lb of tea costing $3.25 per pound. Find the cost of the tea mixture.

47. A pharmacist has 200 milliliters of a solution that is 40% active ingredient. How much pure water should she add to the solution to get a solution that is 25% active ingredient?

48. How many pounds of a 15% aluminum alloy must be mixed with 500 lb of a 22% aluminum alloy to make a 20% aluminum alloy?

49. A silversmith mixed 25 g of a 70% silver alloy with 50 g of a 15% silver alloy. What is the percent concentration of the resulting alloy?

50. How many milliliters of alcohol must be added to 200 ml of a 25% iodine solution to make a 10% iodine solution?

51. A car radiator contains 9 liters of a 40% antifreeze solution. How many liters will have to be replaced with pure antifreeze if the resulting solution is to be 60% antifreeze?

52. Two planes are 1620 mi apart and are traveling toward each other. One plane is traveling 120 mph faster than the other plane. The planes meet in 1.5 h. Find the speed of each plane.

53. An airplane traveling 390 mph in still air encounters a 65-mph headwind. To the nearest minute, how long will it take the plane to travel 725 mi into the wind?

54. Angela leaves from Jocelyn's house on her bicycle traveling at 12 mph. Ten minutes later, Jocelyn leaves her house on her bicycle traveling at 15 mph to catch up with Angela. How long, in minutes, does it take Jocelyn to reach Angela?

55. Hana walked from her home to a bicycle repair shop at a rate of 3.5 mph and then rode her bicycle back home at a rate of 14 mph. If the total time spent traveling was one hour, how far from Hana's home is the repair shop?

56. A jogger and a cyclist set out at 9 A.M. from the same point headed in the same direction. The average speed of the cyclist is four times the average speed of the jogger. In 2 h, the cyclist is 33 mi ahead of the jogger. How far did the cyclist ride?

57. If a 2 mi long parade is proceeding at 3 mph, how long will it take a runner jogging at 6 mph to travel from the front of the parade to the end of the parade?